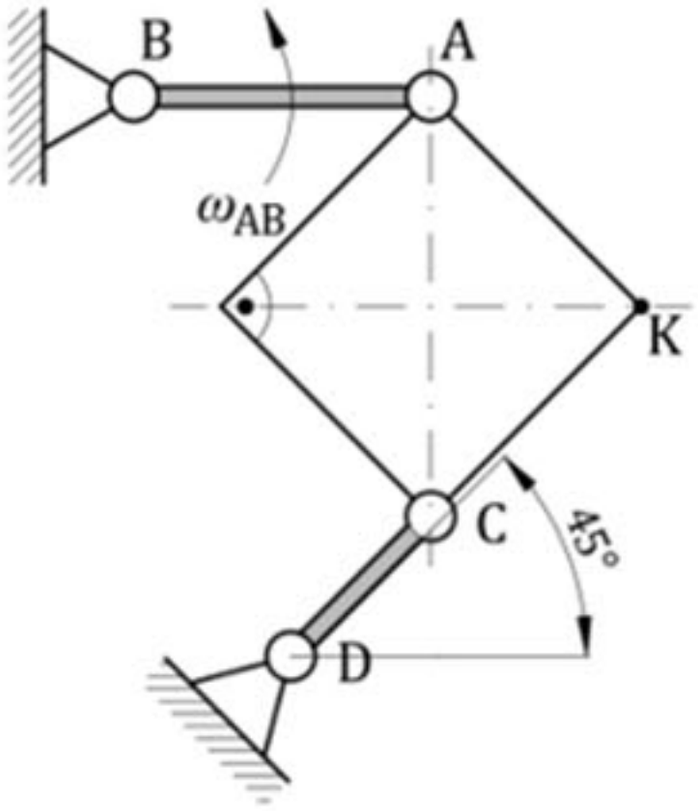
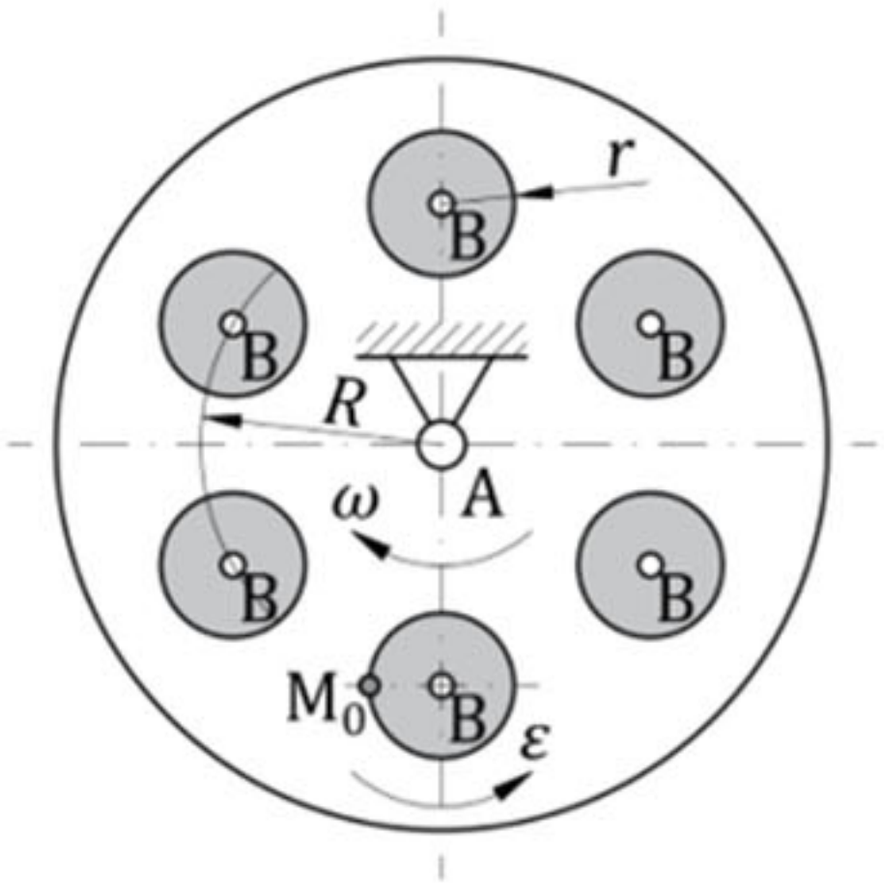
ДРУГИ КОЛОКВИЈУМ ИЗ КИНЕМАТИКЕ

1. Угаона брзина штапа AB мијења се према закону $\omega_{AB} = 4\sin(2t)$. У тренутку $t_* = \pi/8$, механизам заузима положај приказан на слици. Ако је $\overline{AB} = 2\overline{CD} = 0{,}5\sqrt{2}\,\mathrm{m}$ и ако је дужина странице квадрата 1 m, у посматраном временском тренутку одредити брзину и убрзање тачке K.



2. Велика кружна плоча обрће се око непомичног ослонца A константном угаоном брзином ω. За период од пет секунди направи један пун обртај. Истовремено се око осовине B обрћу мали дискови константним угаоним убрзањем $\varepsilon = \pi/8\ \mathrm{s}^{-2}$ у односу на велику плочу и почетном угаоном брзином од $\pi/8\ \mathrm{s}^{-1}$. Одредити апсолутну брзину и убрзање тачке M након двије секунде од почетка кретања, ако је њен почетни положај M_0 приказан на слици. Дато је: $R = 2{,}5$ m и $r = 1$ m.



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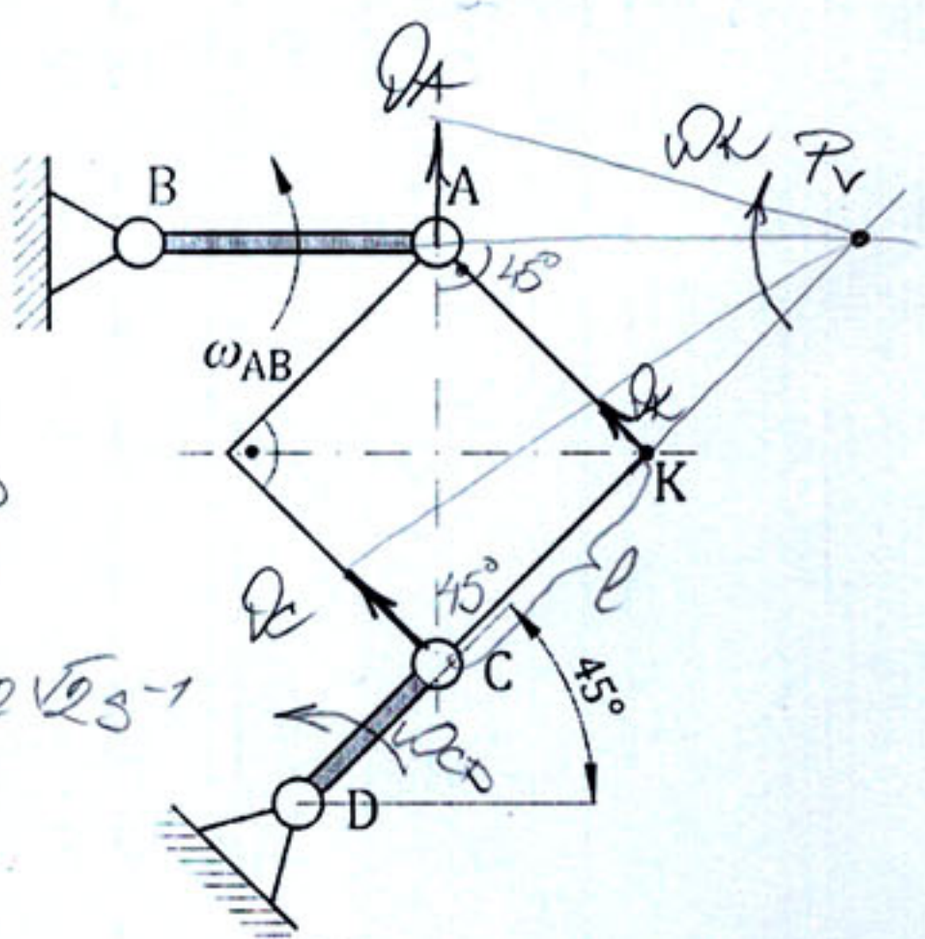


$\omega_{AB}^* = 4 \sin\frac{\pi}{4}$

$= 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}\, s^{-1}$

$\overline{AP_V} = \overline{AC} = \ell\sqrt{2} = \sqrt{2}\, m$

$\overline{CP_V} = 2\ell = 2\, m$

$\overline{KP_V} = \ell = 1\, m$

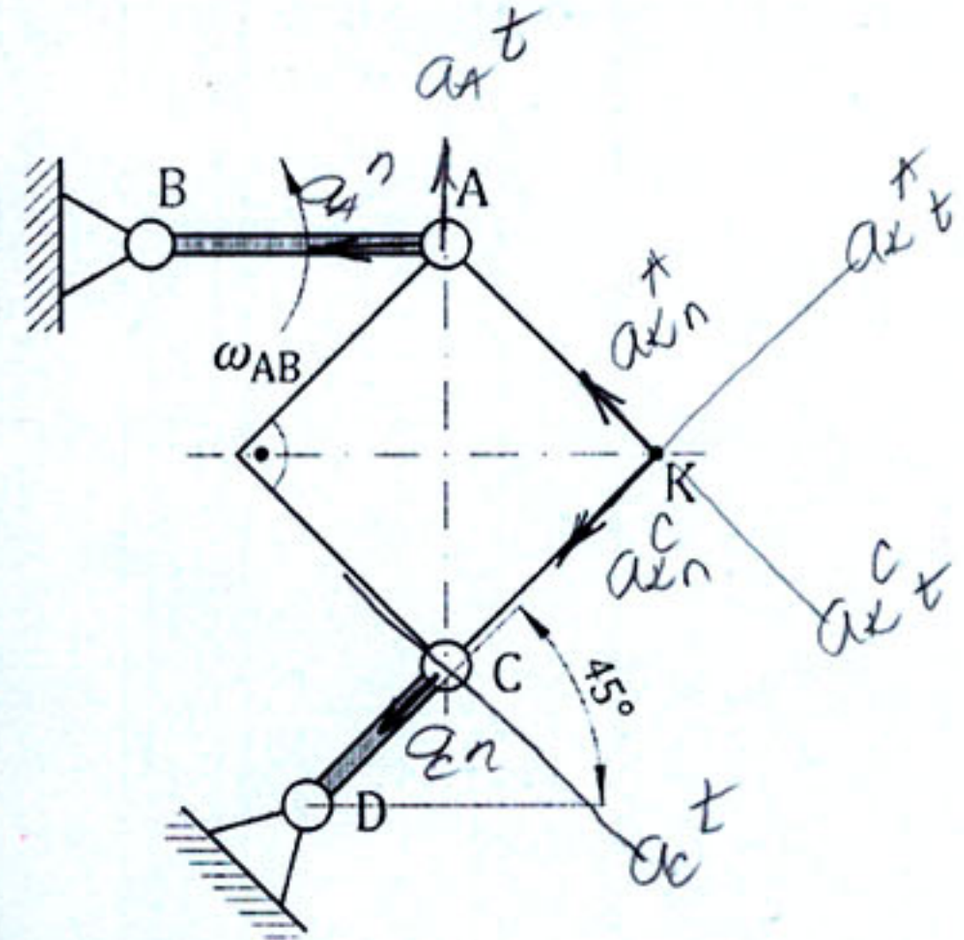
$\vartheta_A = \overline{AB}\,\omega_{AB} = \frac{\sqrt{2}}{2} \cdot 2\sqrt{2} = 2\ m/s$

$\vartheta_A = \overline{AP_V}\,\omega_K \Rightarrow \omega_K = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}\, s^{-1}$

$\underline{\underline{\vartheta_K = \overline{KP_V}\,\omega_K = 1 \cdot \sqrt{2} = \sqrt{2}\ m/s}}$

$\vartheta_C = \overline{CP_V} \cdot \omega_K = 2\sqrt{2}\ m/s$

$\vartheta_C = \overline{CD} \cdot \omega_{CD} \Rightarrow \omega_{CD} = \frac{2\sqrt{2}}{0{,}25\sqrt{2}} = 8\, s^{-1}$



$\varepsilon_{AB} = 8\cos(2t)$

$\varepsilon_{AB}^* = 8 \cdot \frac{\sqrt{2}}{2} = 4\sqrt{2}\, s^{-2}$

$\vec{a}_K = \vec{a}_A + \vec{a}_{Kt}^A + \vec{a}_{Kn}^A$

$\vec{a}_K = \vec{a}_C + \vec{a}_{Kt}^C + \vec{a}_{Kn}^C$

$a_A^t = \overline{AB}\,\varepsilon_{AB} = \frac{\sqrt{2}}{2} \cdot 4\sqrt{2} = 4$

$a_A^n = \overline{AB}\,\omega_{AB}^2 = \frac{\sqrt{2}}{2} \cdot 8 = 4\sqrt{2}$

$a_{Kn}^A = \overline{KA} \cdot \omega_K^2 = 1 \cdot 2 = 2$

$a_C^n = CD \cdot \omega_{CD}^2 = \frac{\sqrt{2}}{4} \cdot 64 = 16\sqrt{2}$

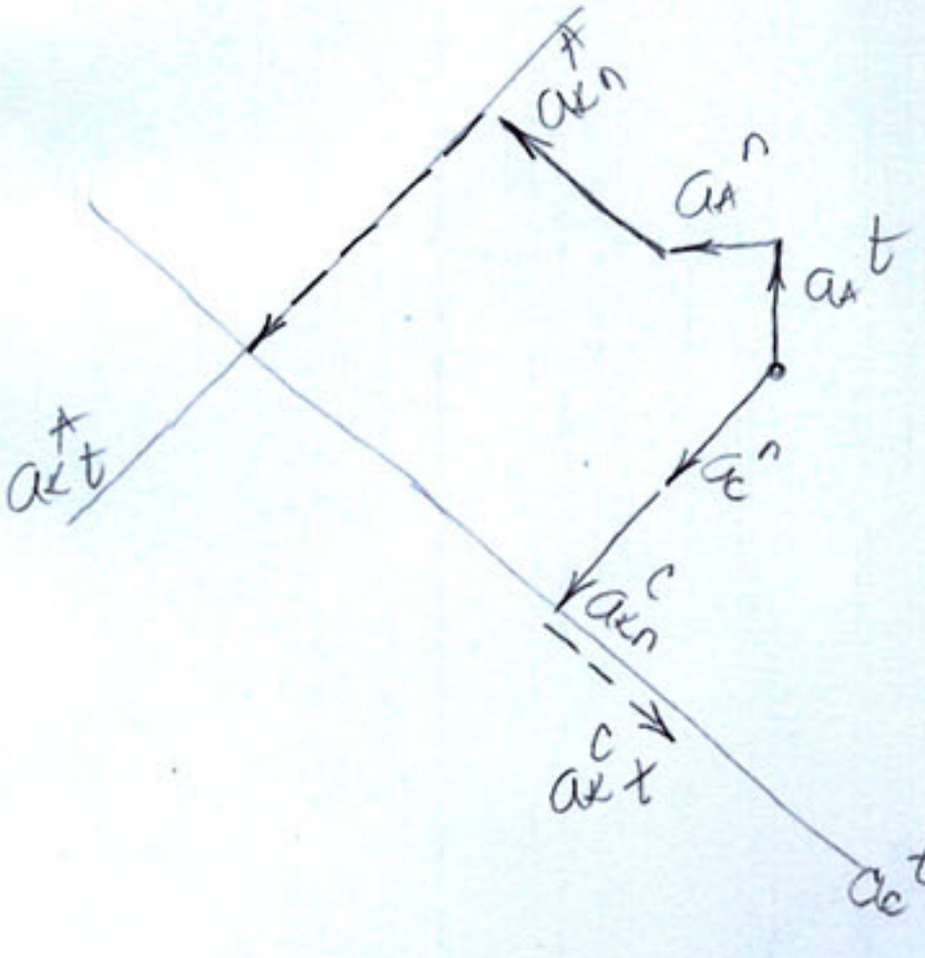
$a_{Kn}^C = \overline{KC} \cdot \omega_K^2 = 1 \cdot 2 = 2$

$a_{Kt}^A = \overline{KA} \cdot \varepsilon_K = \varepsilon_K$

$a_{Kt}^C = \overline{KC} \cdot \varepsilon_K = \varepsilon_K$

$a_C^t = CD \cdot \varepsilon_{CD}$

$\vec{a}_A^t + \vec{a}_A^n + \vec{a}_{Kt}^A + \vec{a}_{Kn}^A = \vec{a}_C^t + \vec{a}_C^n + \vec{a}_{Kt}^C + \vec{a}_{Kn}^C$



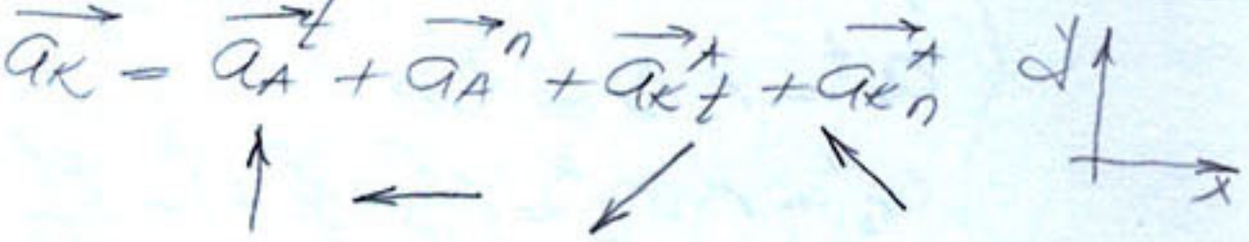
$a_{Kt}^A = a_{Kn}^C + a_C^n + a_A^t \cdot \frac{\sqrt{2}}{2} - a_A^n \frac{\sqrt{2}}{2}$

$= 2 + 16\sqrt{2} + 2\sqrt{2} - 4 = 18\sqrt{2} - 2 = 23{,}46$

$a_{Kt}^A = 1 \cdot \varepsilon_K \Rightarrow \varepsilon_K = 18\sqrt{2} - 2$

$a_{Kt}^C = 1 \cdot \varepsilon_K = 18\sqrt{2} - 2$

$\vec{a}_K = \vec{a}_A^t + \vec{a}_A^n + \vec{a}_{Kt}^A + \vec{a}_{Kn}^A$



$a_{Kx} = -a_A^n - a_{Kt}^A \frac{\sqrt{2}}{2} - a_{Kn}^A \frac{\sqrt{2}}{2}$

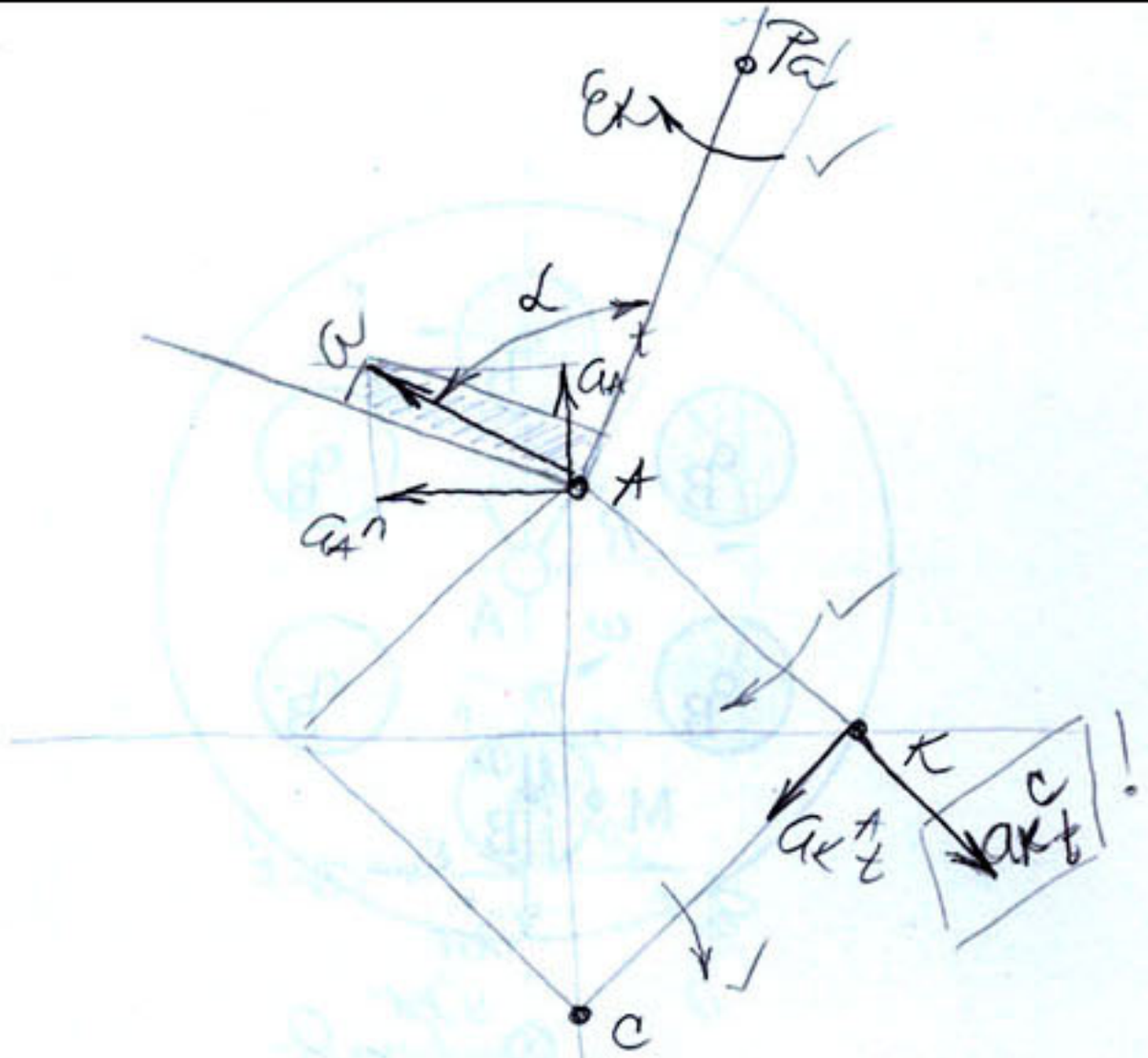
$= -4\sqrt{2} - (18\sqrt{2}-2)\frac{\sqrt{2}}{2} - 2\frac{\sqrt{2}}{2}$

$= -4\sqrt{2} - 18 + \sqrt{2} - \sqrt{2}$

$a_{Ky} = a_A^t - a_{Kt}^A \frac{\sqrt{2}}{2} + a_{Kn}^A \frac{\sqrt{2}}{2} = 4 - (18\sqrt{2}-2)\frac{\sqrt{2}}{2} + 2\frac{\sqrt{2}}{2}$

$= 4 - 18 + \sqrt{2} + \sqrt{2} = 2\sqrt{2} - 14$

$\underline{a_K} = \sqrt{(-4\sqrt{2}-18)^2 + (2\sqrt{2}-14)^2} = \underline{26{,}16}\ м/с^2$



$\operatorname{tg}\alpha = \frac{\varepsilon}{\omega^2} = 11{,}73$

$\alpha = 85°$

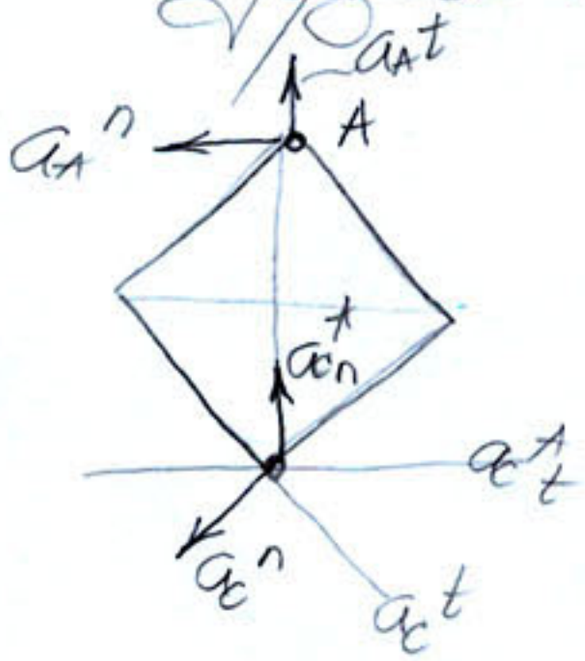
Задача решена С (1 раз)

$\vec{a}_C^t = \vec{a}_{Kn}^A + \vec{a}_A^n \frac{\sqrt{2}}{2} + \vec{a}_A^t \frac{\sqrt{2}}{2} + \vec{a}_{Kt}^C$

$= 4 + 2\sqrt{2} + 18\sqrt{2} = 4 + 20\sqrt{2}$

$\underline{a_C} = \sqrt{a_{Ct}^2 + a_{Cn}^2} = \sqrt{(40+20\sqrt{2})^2 + 256 \cdot 2} = \underline{71{,}94}\ м/с^2$

Задача также С (II способ)



$a_{Cn}^A = AC \cdot \omega^2 = \sqrt{2} \cdot 2 = 2\sqrt{2}\ м/с^2$

$\vec{a}_C^t + \vec{a}_C^n = \vec{a}_A^t + \vec{a}_A^n + \vec{a}_{Ct}^A + \vec{a}_{Cn}^A$

$y: -a_C^t \frac{\sqrt{2}}{2} - a_C^n \frac{\sqrt{2}}{2} = a_A^t + a_{Cn}^A$

$a_C^t = \frac{-2}{\sqrt{2}}\left(a_C^n \frac{\sqrt{2}}{2} + a_A^t + a_{Cn}^A\right)$

$= -\sqrt{2}\left(16\sqrt{2}\frac{\sqrt{2}}{2} + 4 + 2\sqrt{2}\right) = -(20\sqrt{2}+4)$ — верно, другой способ

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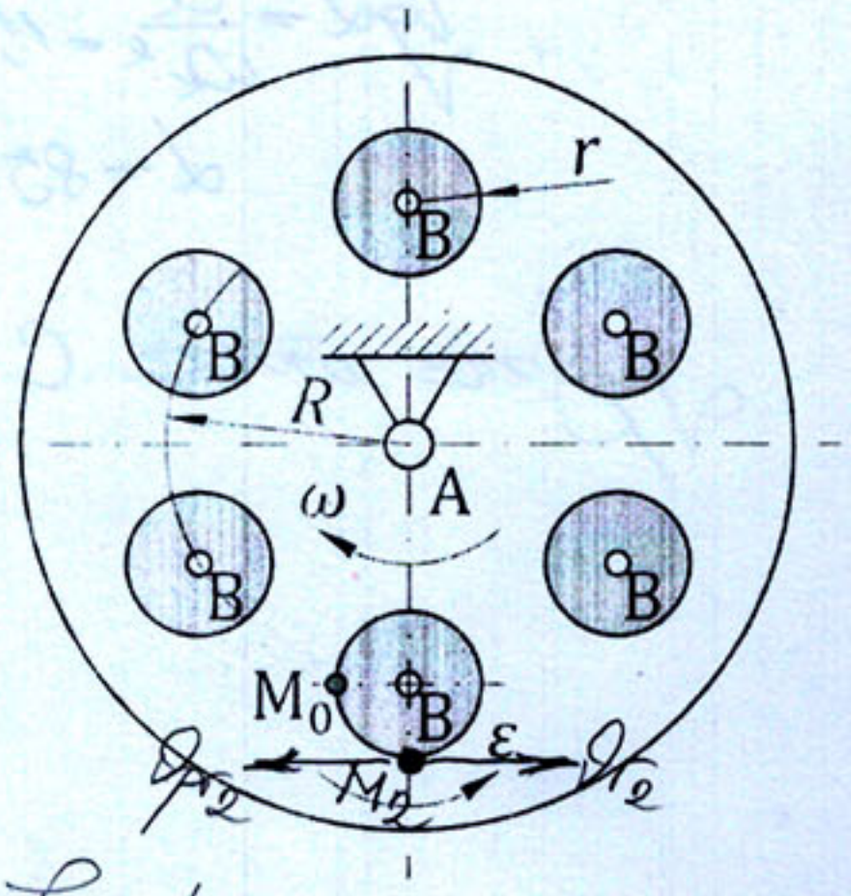


$$\omega_{pr} = \frac{2\pi\ rad}{5\ s}$$

$$\varepsilon_r = \pi/8 = const \rightarrow \omega_r = \omega_{r_0} + \varepsilon_r t$$

$$\omega_r = \frac{\pi}{8} + \frac{\pi}{8} t$$

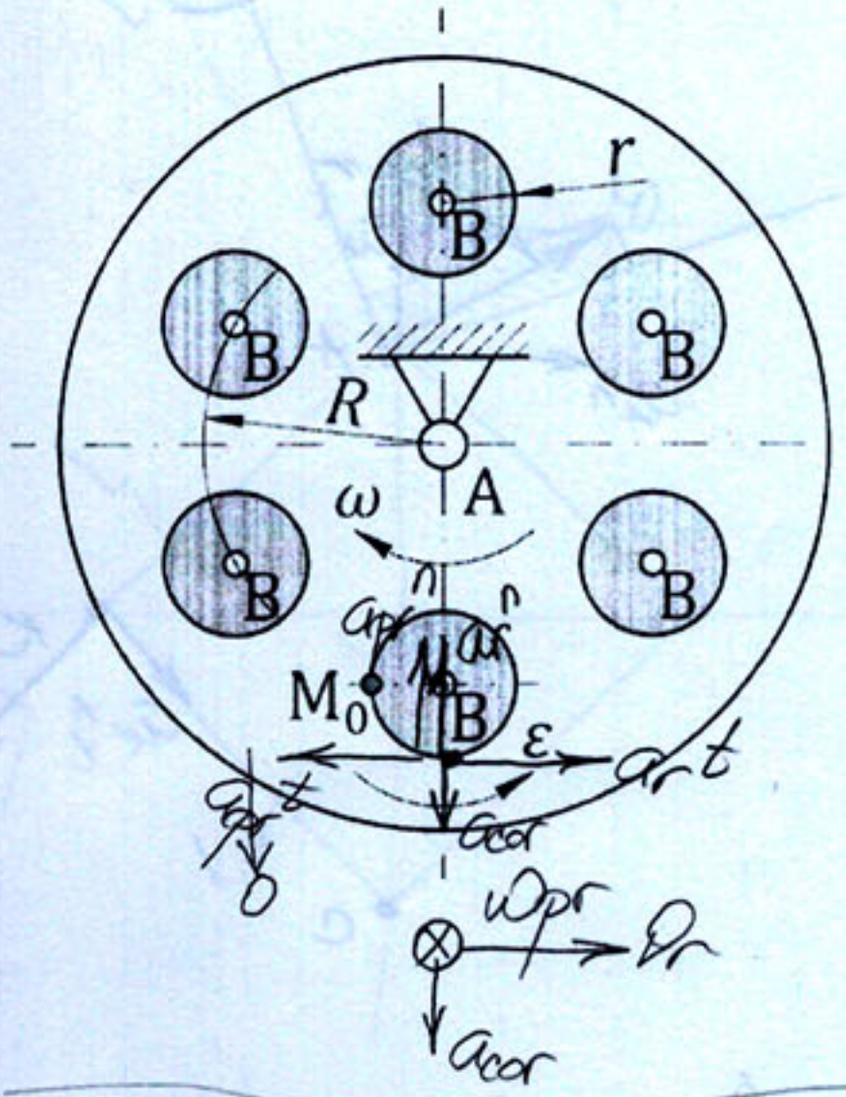
$$\varphi_r = \frac{\pi}{8} t + \frac{\pi}{8} \frac{t^2}{2}$$

$$\varphi_{r_2} = \frac{\pi}{8} \cdot 2 + \frac{\pi}{8} \cdot \frac{4}{2} = \frac{\pi}{2}\ rad$$

$$v_{pr_2} = (R+r) \cdot \omega_{pr_2} = 3{,}5 \cdot \frac{2}{5}\pi = 1{,}4\pi = \underline{4{,}4\ m/s}$$

$$v_{r_2} = r \cdot \omega_{r_2} = 1 \cdot \left(\frac{\pi}{8} + \frac{\pi}{8} \cdot 2\right) = \frac{3}{8}\pi = \underline{1{,}18\ m/s}$$

$$v_{a_2} = v_{pr_2} - v_{r_2} = \left(\frac{7}{5} - \frac{3}{8}\right)\pi = \pi\frac{56-15}{40} = \frac{41}{40}\pi = 3{,}22\ m/s$$



$$a_{pr}^{t} = 0$$

$$a_{pr}^{n} = (R+r)\omega_{pr}^2 = 3{,}5\left(\frac{2\pi}{5}\right)^2 = \frac{14}{25}\pi^2$$

$$a_r^{t} = r \cdot \varepsilon_r = \pi/8$$

$$a_{r_2}^{n} = r \cdot \omega_{r_2}^2 = \left(\frac{\pi}{8} + \frac{\pi}{8} \cdot 2\right)^2 = \frac{9\pi^2}{64}$$

$$a_{cor_2} = 2 \cdot \frac{2\pi}{5} \cdot \frac{3}{8}\pi \cdot \sin 90^\circ = \frac{3}{10}\pi^2$$

$$\underline{a_{a_2}} = \sqrt{\left(\pi/8\right)^2 + \left(\frac{14}{25}\pi^2 + \frac{9\pi^2}{64} - \frac{3}{10}\pi^2\right)^2} = \underline{3{,}97\ m/s^2}$$