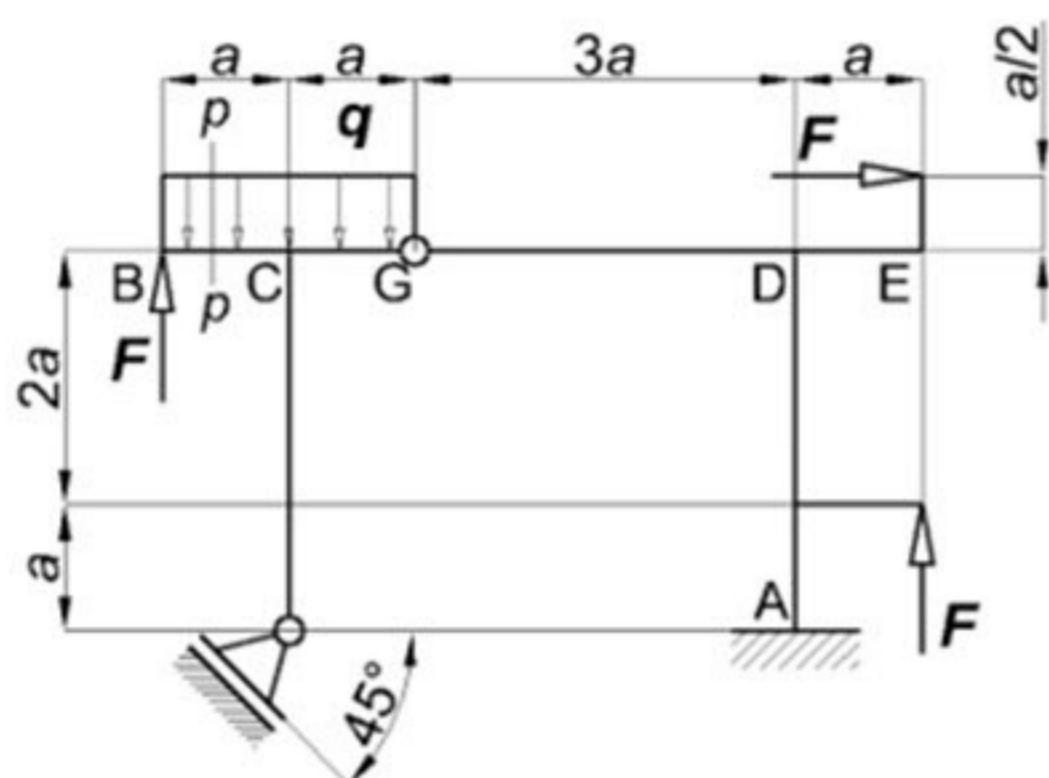
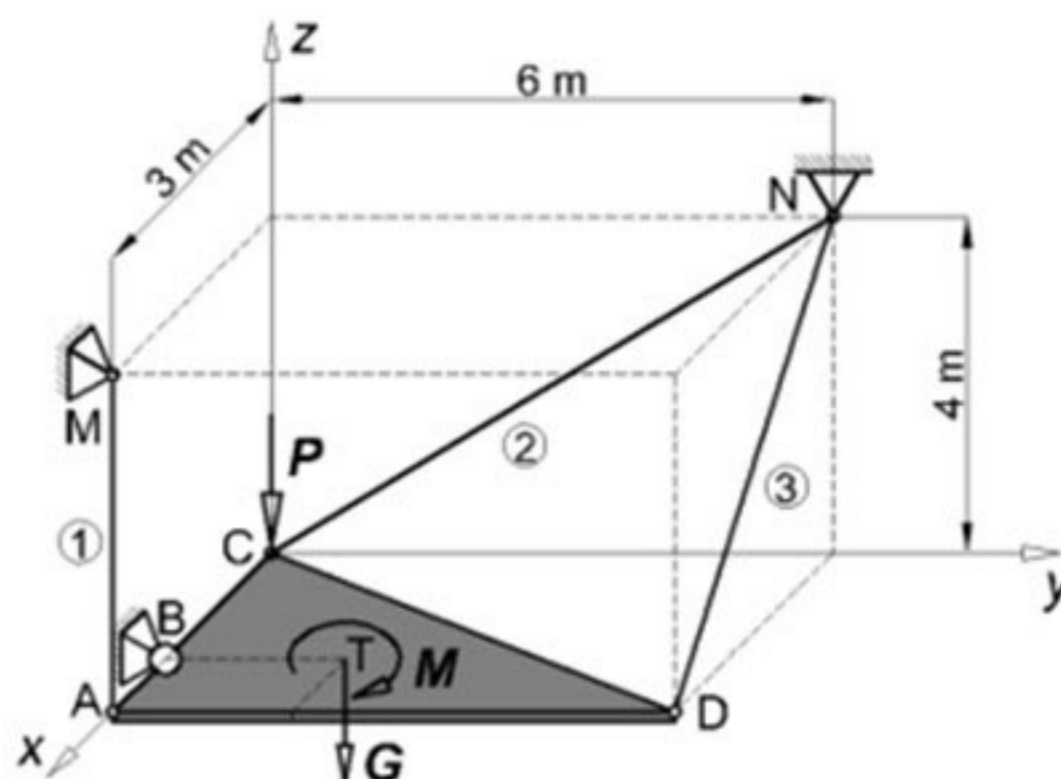


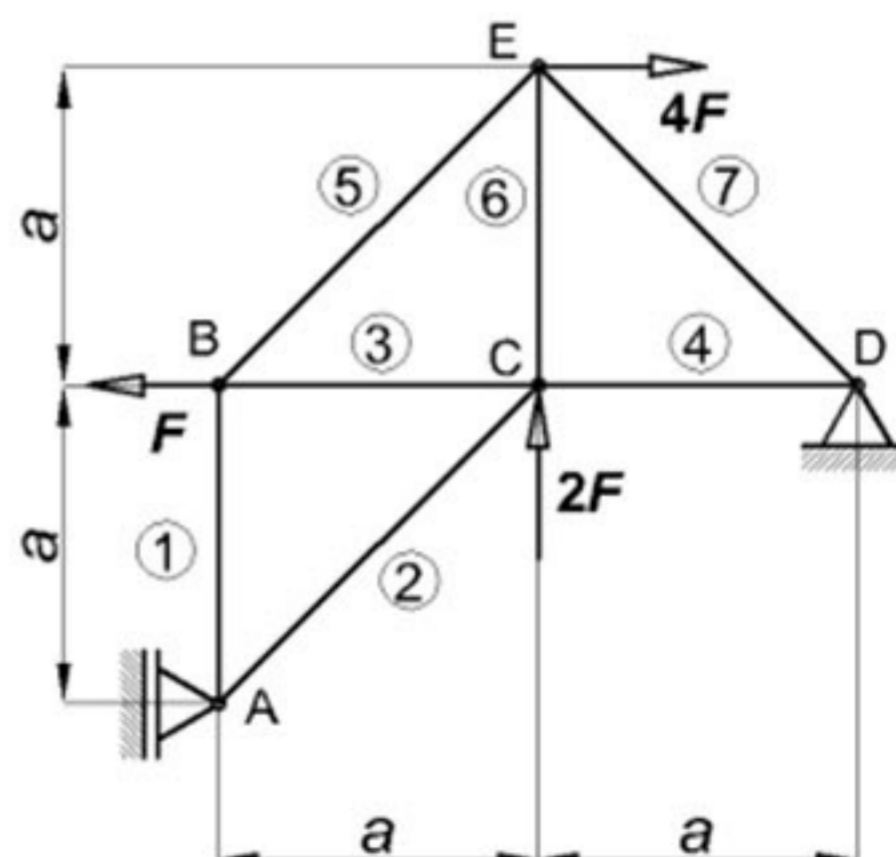
**ЗАВРШНИ ИСПИТ ИЗ СТАТИКЕ**

1. Одредити реакције веза хомогене плоче тежине  $G = 2 \text{ kN}$  приказане на слици. На плочу у тачки С дјелује сила  $P$  интензитета  $8 \text{ kN}$ . У тачки В је плоча везана за сферни зглоб, а у тачкама А, С и D за лаке круте штапове. У равни плоче дјелује момент  $M$  интензитета  $6 \text{ kNm}$ .



2. Аналитички одредити отпоре ослонаца/укљештења рама приказаног на слици и нацртати статичке дијаграме, ако је  $F = 4 \text{ kN}$ ,  $q = 4 \text{ kN/m}$  и  $a = 2 \text{ m}$ . Израчунати момент савијања и трансферзалну силу у пресеку  $p \div p$ .

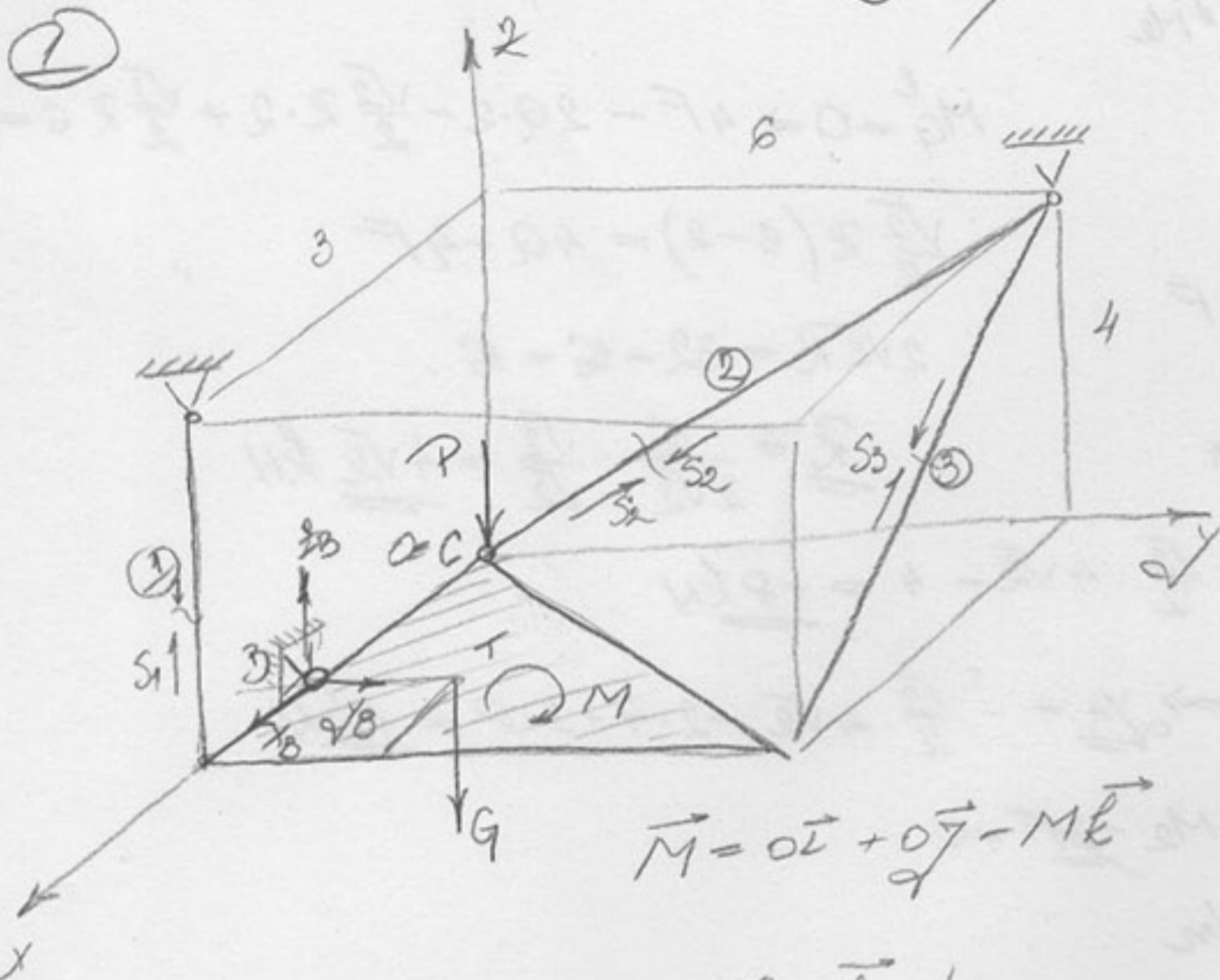
3. Одредити реакције ослонаца раванског решеткастог носача приказаног на слици. Потом одредити силе у штаповима Кремонином методом и утврдити врсту оптерећења којем су штапови изложени. Добијене резултате провјерити Ритеровом методом за штапове 2, 3 и 5. Дато је:  $F = 4 \text{ kN}$  и  $a = 0,5 \text{ m}$ .



Предметни наставник:  
Проф. др Оливера Јовановић

Сарадник:  
Раде Грујићић

# Статика - забрания услови



$$\vec{r}_B = x_B \vec{i} + y_B \vec{j} + z_B \vec{k}$$

$$\vec{S}_1 = 0 \vec{i} + 0 \vec{j} + S_1 \vec{k}$$

$$\vec{S}_2 = 0 \vec{i} + \frac{S_2 \cdot 6}{\sqrt{6^2+4^2}} \vec{j} + \frac{S_2 \cdot 4}{\sqrt{6^2+4^2}} \vec{k}$$

$$= 0 \vec{i} + \frac{6 S_2}{\sqrt{52}} \vec{j} + \frac{4 S_2}{\sqrt{52}} \vec{k}$$

$$\vec{S}_3 = -\frac{3 S_3}{\sqrt{3^2+4^2}} \vec{i} + 0 \vec{j} + \frac{4 S_3}{\sqrt{3^2+4^2}} \vec{k}$$

$$= -\frac{3 S_3}{5} \vec{i} + 0 \vec{j} + \frac{4 S_3}{5} \vec{k}$$

$$\vec{P} = 0 \vec{i} + 0 \vec{j} - P \vec{k}$$

$$\vec{G} = 0 \vec{i} + 0 \vec{j} - G \vec{k}$$

$$\vec{M} = 0 \vec{i} + 0 \vec{j} - M \vec{k}$$

$$\vec{M}_C^{\vec{r}_B} = \vec{r}_B \times \vec{S}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_B & y_B & z_B \\ 0 & 0 & S_2 \end{vmatrix} = 0 \vec{i} - 2 z_B \vec{j} + 2 y_B \vec{k}$$

$$\vec{M}_C^{\vec{S}_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & S_1 \\ 0 & 0 & 0 \end{vmatrix} = 0 \vec{i} - 3 S_1 \vec{j} + 0 \vec{k}$$

$$\vec{M}_C^{\vec{S}_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0 \vec{i} + 0 \vec{j} + 0 \vec{k}$$

$$\vec{M}_C^{\vec{S}_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{3}{5} S_3 & 0 & \frac{4}{5} S_3 \\ 0 & 0 & 0 \end{vmatrix} = \frac{24}{5} S_3 \vec{i} - \frac{12}{5} S_3 \vec{j} + \frac{12}{5} S_3 \vec{k}$$

$$\vec{M}_C^{\vec{P}} = 0 \vec{i} + 0 \vec{j} + 0 \vec{k}$$

$$\vec{M}_C^{\vec{G}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -G \\ 0 & 0 & 0 \end{vmatrix} = -2 G \vec{i} + 2 G \vec{j} + 0 \vec{k}$$

$$F_{xy} = 0 \Rightarrow 2 y_B + \frac{6 S_2}{\sqrt{52}} = 0$$

$$S_2 = -\frac{\sqrt{52}}{6} y_B = -\frac{2\sqrt{13}}{8} \cdot \frac{3}{2} = -\frac{\sqrt{13}}{2}$$

(упреждак)  $= -1,8 \text{ kN}$

$$F_{xz} = 0 \Rightarrow z_B + S_1 + \frac{4 S_2}{\sqrt{52}} + \frac{4}{5} S_3 - P - G = 0$$

$$S_1 + 1 - \frac{3}{2} S_1 - \frac{4 \cdot \sqrt{13}}{2 \sqrt{52}} + \frac{4}{5} \cdot \frac{3}{2} - 8 - 2 = 0$$

$$\frac{1}{2} S_1 - 1 + \frac{3}{2} = 10 = 0$$

$$\frac{S_1}{2} = \frac{2}{3} - \frac{30}{3} = -\frac{28}{3} \Rightarrow S_1 = -\frac{56}{3}$$

$$S_2 = -\frac{56}{3} = -18,67 \text{ kN}$$

(упреждак)

(устезање)

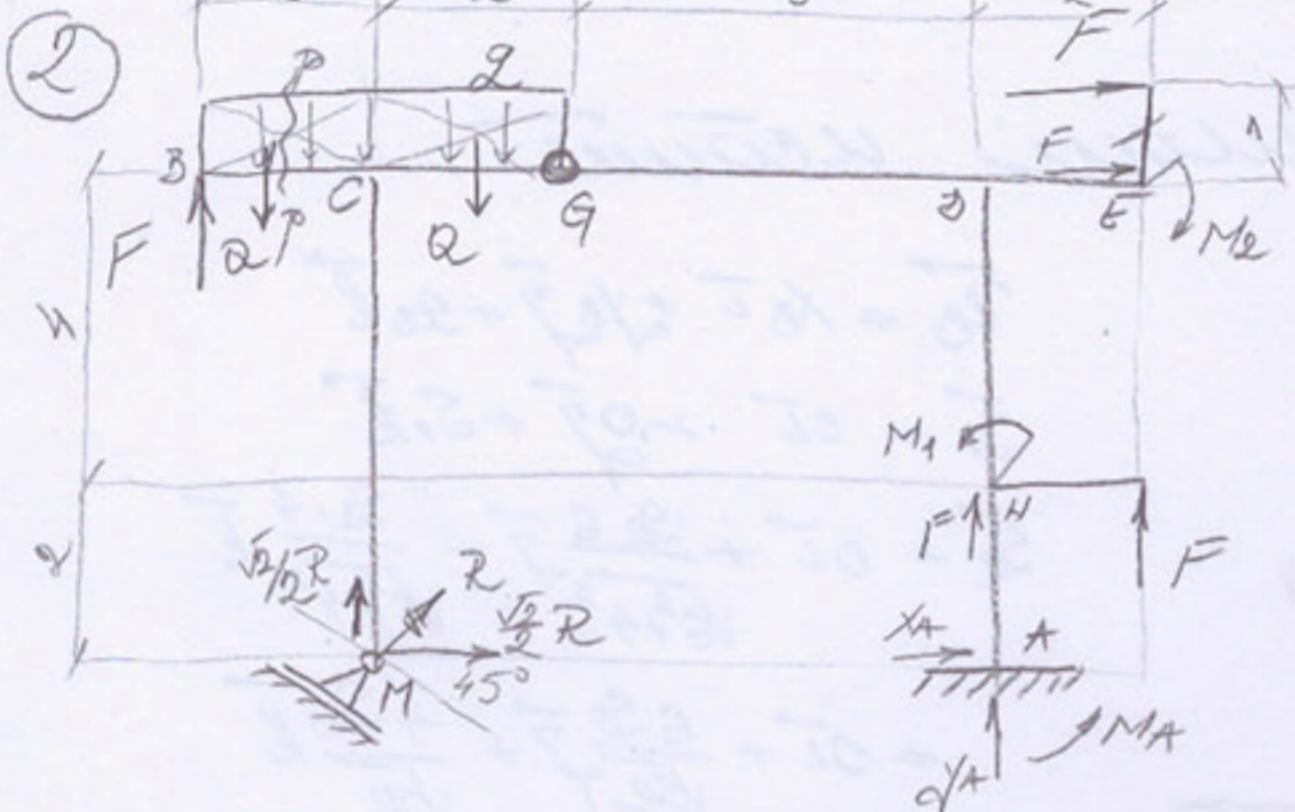
$$M_{rx} = 0 \Rightarrow \frac{24}{5} S_3 - 2G = 0 \Rightarrow S_3 = \frac{10G}{24} = \frac{20}{24} = \frac{5}{6} \text{ kN} = 0,83 \text{ kN}$$

$$M_{ry} = 0 \Rightarrow -2 z_B - 3 S_1 - \frac{12}{5} S_3 + 2G = 0 \Rightarrow z_B = -\frac{3}{2} S_1 - \frac{6}{5} \cdot \frac{5}{6} + 2 = 1 - \frac{3}{2} S_1$$

$$M_{rz} = 0 \Rightarrow 2 y_B + \frac{12}{5} S_3 - M = 0 \Rightarrow y_B = -\frac{9}{5} S_3 + \frac{M}{2} = -\frac{9}{5} \cdot \frac{5}{6} + \frac{6}{2} = -\frac{3}{2} + \frac{6}{2} = \frac{3}{2} \text{ kN} = 1,5 \text{ kN}$$

$$F_{rx} = 0 \Rightarrow x_B - \frac{3}{5} S_3 = 0 \Rightarrow x_B = \frac{3}{5} S_3 = \frac{3}{5} \cdot \frac{5}{6} = \frac{1}{2} = 0,5 \text{ kN}$$

$$z_B = 1 - \frac{3}{2} S_1 = 1 + \frac{3}{2} \cdot \frac{56}{3} = 28 \text{ kN}$$



$$M_1 = 2F = 8 \text{ kNm} \quad M_2 = 1F = 4 \text{ kNm}$$

$$Q = 2q = 8 \text{ kN}$$

$$M_G^L = 0 \Rightarrow 4F - 2Q \cdot 2 + \frac{\sqrt{2}}{2} R \cdot 2 - \frac{\sqrt{2}}{2} R \cdot 6 = 0$$

$$-\frac{\sqrt{2}}{2} R (6-2) = 4Q - 4F$$

$$-2\sqrt{2} R = 32 - 16 = 16$$

$$R = \frac{16}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -4\sqrt{2} \text{ kN}$$

$$\sum X_i = 0 \Rightarrow \frac{\sqrt{2}}{2} R + X_A + F = 0 \Rightarrow X_A = +\frac{\sqrt{2}}{2} \cdot 4\sqrt{2} - 4 = 0 \text{ kN}$$

$$\sum Y_i = 0 \Rightarrow \frac{\sqrt{2}}{2} R + Y_A + F + F - 2Q = 0 \Rightarrow Y_A = +\frac{\sqrt{2}}{2} \cdot 4\sqrt{2} - 2 \cdot 4 + 2 \cdot 8 = 10 \text{ kN}$$

$$\sum M_M = 0 \Rightarrow 8Y_A + M_A + 2F + M_1 - 6F - M_2 - 2F = 0$$

$$M_A = -8 \cdot 10 - 8 + 4 = -100 \text{ kNm}$$

$$M_M^L = 0$$

$$M_C^L = -\frac{\sqrt{2}}{2} R \cdot 6 = +\frac{\sqrt{2}}{2} \cdot 4\sqrt{2} \cdot 6 = +24 \text{ kNm}$$

$$M_B^L = 0$$

$$M_C^R = 2F - Q \cdot 1 = 8 - 8 = 0 \text{ kNm}$$

$$M_C^L = 2F - Q \cdot 1 - \frac{\sqrt{2}}{2} R \cdot 6 = 0 + 24 = +24 \text{ kNm}$$

$$M_E^d = -M_2 = -4 \text{ kNm}$$

$$M_E^d = 0$$

$$M_D^d = M_A + 6X_A + M_1 = -100 - 6 \cdot 0 + 8 = -92 \text{ kNm}$$

$$M_D^L = -\frac{\sqrt{2}}{2} R \cdot 6 + \frac{\sqrt{2}}{2} R \cdot 8 + 10F - 2Q \cdot 8$$

$$= -\frac{\sqrt{2}}{2} \cdot 4\sqrt{2} \cdot 2 + 40 - 16 \cdot 8 = -96 \text{ kNm}$$

$$M_D^d = -M_2 + M_1 + M_A + 6X_A$$

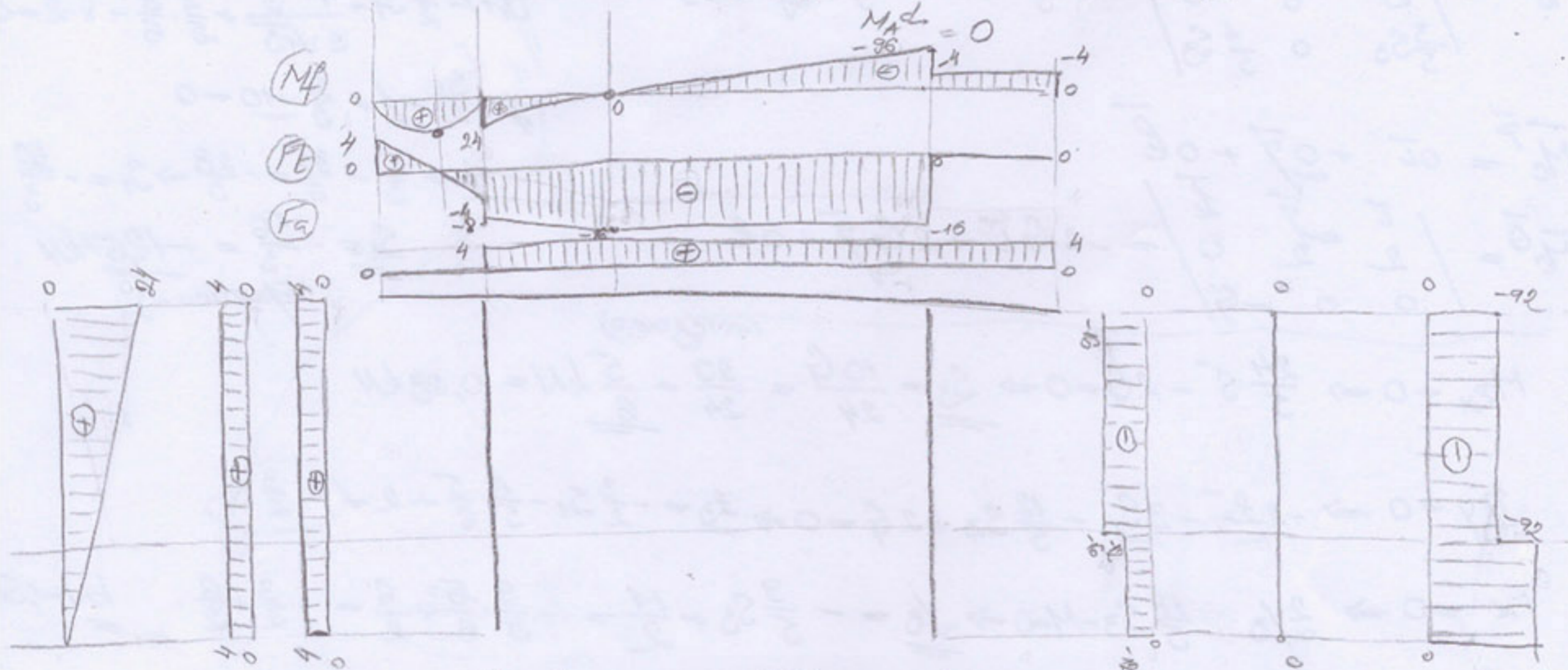
$$= -4 + 8 - 100 - 6 \cdot 0 = -96 \text{ kNm}$$

$$M_D^d = -M_2 = -4 \text{ kNm}$$

$$M_N^d = M_1 + M_A + 2X_A = 8 - 100 + 2 \cdot 0 = -92 \text{ kNm}$$

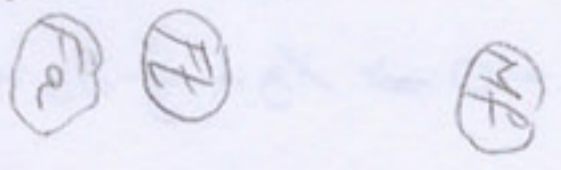
$$M_N^d = M_A + 2X_A = -100 + 2 \cdot 0 = -100 \text{ kNm}$$

$$M_A^d = M_A = -100 \text{ kNm}$$

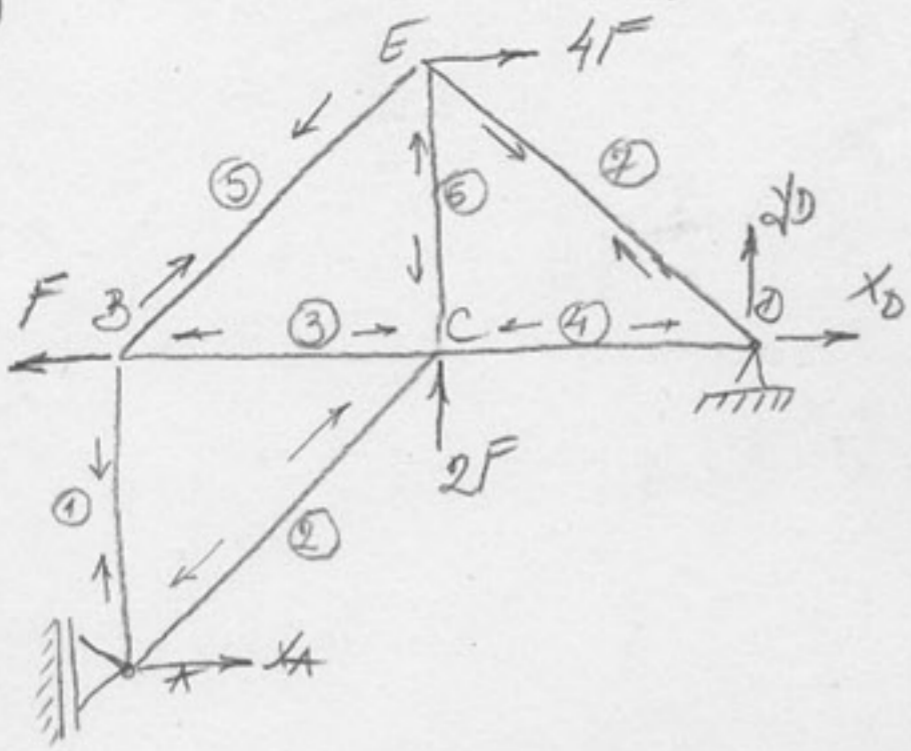


$$F_{FP} = F - q \cdot z \quad F_{FP} = 0 \Rightarrow z = \frac{F}{q} = 1 \text{ m}$$

$$M_{FP} = F \cdot z - q \cdot z \cdot \frac{z}{2} \quad M_{FP_{max}} = 4 \cdot 1 - 4 \cdot 1 \cdot \frac{1}{2} = 2 \text{ kNm}$$



3



$$\sum M_D = 0 \Rightarrow X_A \cdot a - 2Fa - 4Fa = 0$$

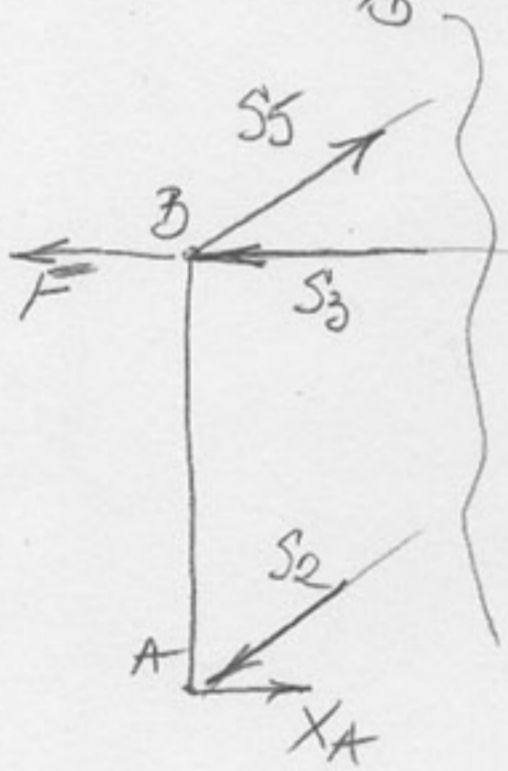
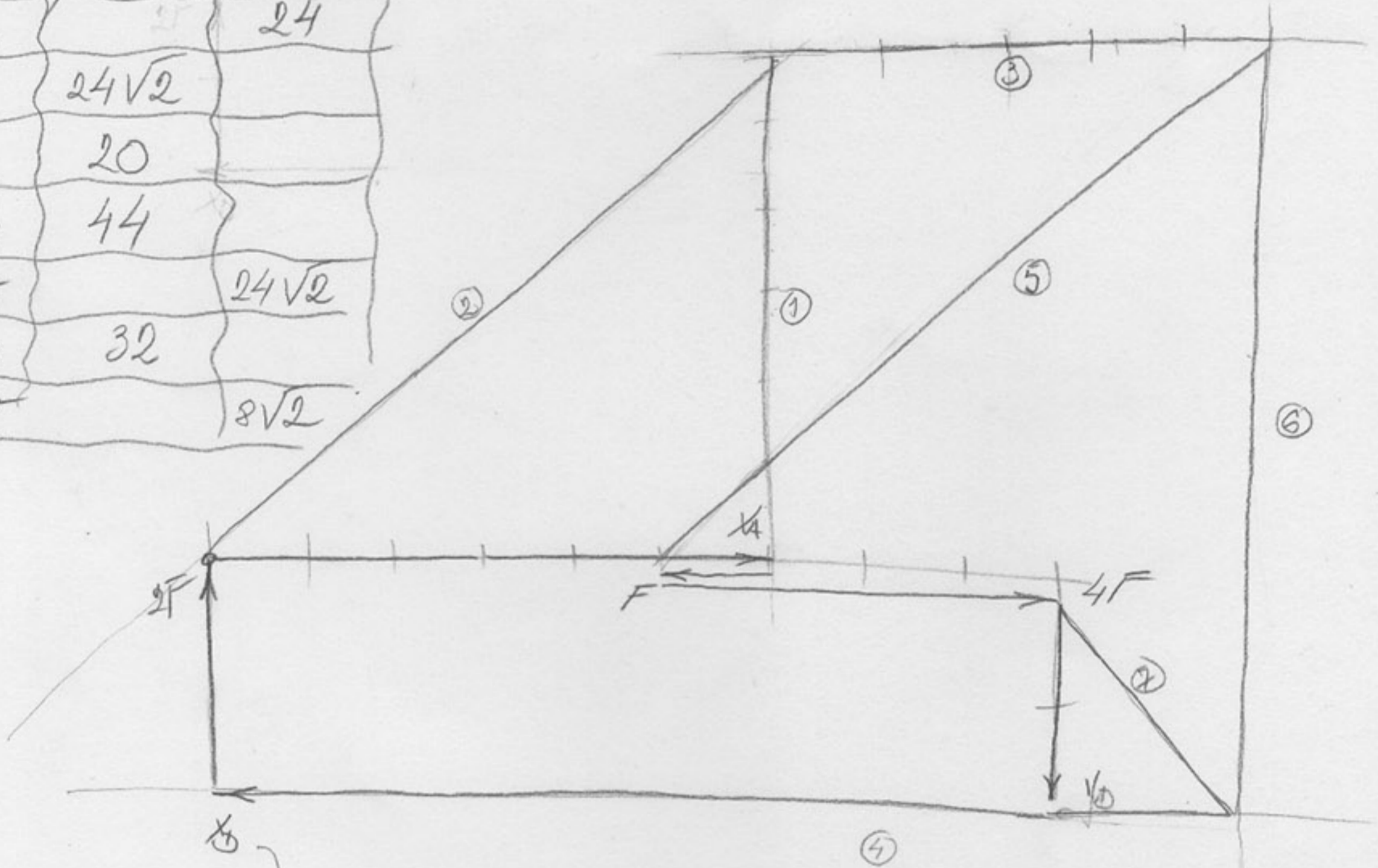
$$\underline{X_A = 6F = 24 \text{ kN}}$$

$$\sum Y_i = 0 \Rightarrow Y_D + 2F = 0 \Rightarrow \underline{Y_D = -2F = -8 \text{ kN}}$$

$$\sum X_i = 0 \Rightarrow X_A + X_D + 4F - F = 0$$

$$\underline{X_D = -24 - 16 + 4 = -36 \text{ kN}}$$

kN	внутренние усилия	
S <sub>1</sub>		24
S <sub>2</sub>	24√2	
S <sub>3</sub>	20	
S <sub>4</sub>	44	
S <sub>5</sub>		24√2
S <sub>6</sub>	32	
S <sub>7</sub>		8√2



$$\sum M_B = 0 \Rightarrow X_A \cdot a - \frac{S_2}{\sqrt{2}} \cdot a = 0 \Rightarrow \underline{S_2 = \sqrt{2} X_A = 24\sqrt{2}}$$

$$\sum Y_i = 0 = \frac{S_5}{\sqrt{2}} - \frac{S_2}{\sqrt{2}} = 0 \Rightarrow \underline{S_5 = S_2 = 24\sqrt{2}}$$

$$\sum X_i = 0 \Rightarrow -F - S_3 + \frac{S_5}{\sqrt{2}} - \frac{S_2}{\sqrt{2}} + X_A = 0$$

$$\underline{S_3 = -4 + \frac{1}{\sqrt{2}} 24\sqrt{2} - \frac{1}{\sqrt{2}} 24\sqrt{2} + 24 = 20 \text{ kN}}$$