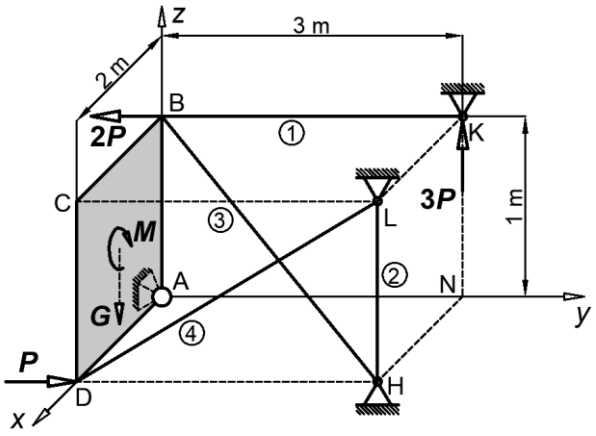
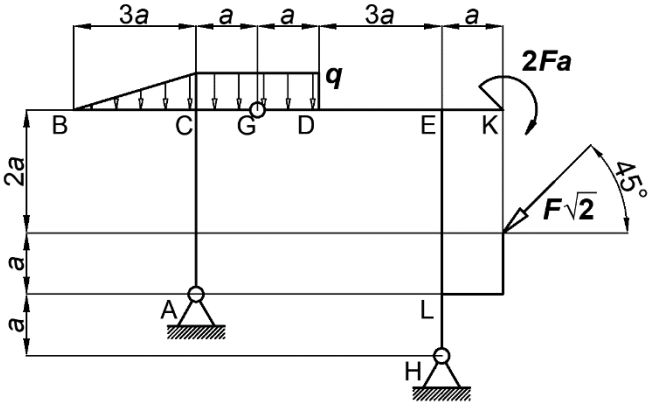
ЗАВРШНИ ИСПИТ ИЗ СТАТИКЕ

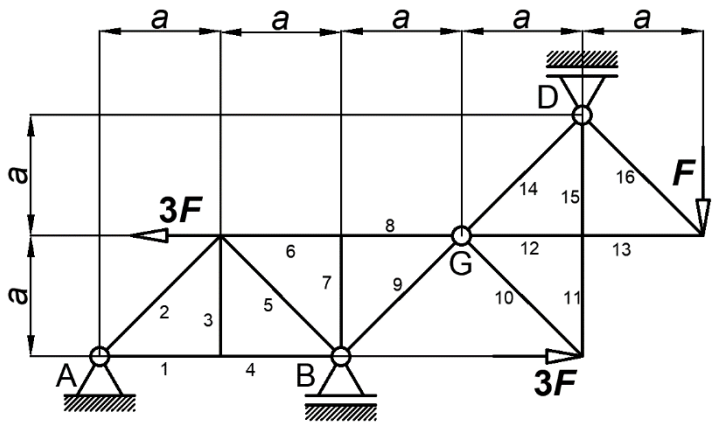
1. Одредити реакције веза хомогене плоче тежине $G = 4$ kN приказане на слици. На плочу дјелује сила P интензитета $3\sqrt{10}$ kN и сила $2P$. У равни плоче дјелује момент M интензитета 1 kNm, чији је смјер дејства приказан на слици. Плоча је у тачки A везана за сферни зглоб, а у тачкама D и B за лаке круте штапове.



2. Одредити реакције ослонаца рама приказаног на слици, а потом нацртати статичке дијаграме ако је $F = 4$ kN, $a = 1$ m и $q = 2$ kN/m. Одредити функције промјене унутрашњих сила на сегменту иза Герберовог зглоба.



3. Одредити силе у штаповима решеткастог носача приказаног на слици Кремонином методом и врсту оптерећења којем су штапови изложени ако је $F = 4$ kN и $a = 1$ m. Добијене резултате провјерити Ритеровом методом за штапове 4, 5, 7 и 8.



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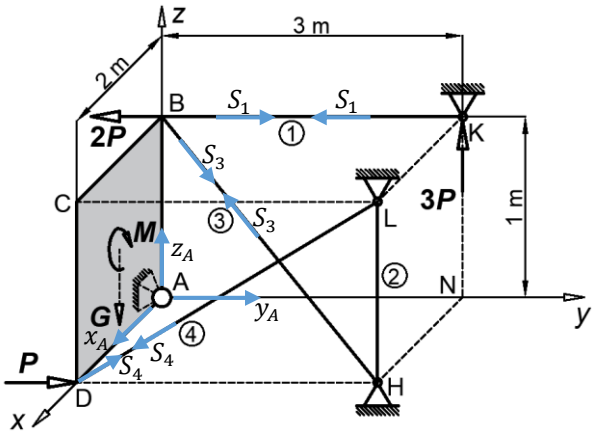
ПРВИ ЗАДАТАК



Напомена: Сила $3\vec{P}$ не дјелује на плочу, а штап 2 за плочу није везан.

$$\vec{P} = P\vec{j} = 3\sqrt{10}\vec{j}$$

$$2\vec{P} = -2P\vec{j} = -6\sqrt{10}\vec{j}$$

$$\vec{G} = -G\vec{k} = -4\vec{k}$$

$$\vec{R}_A = x_A\vec{\imath} + y_A\vec{j} + z_A\vec{k}$$

$$\vec{S}_1 = S_1\vec{j}$$

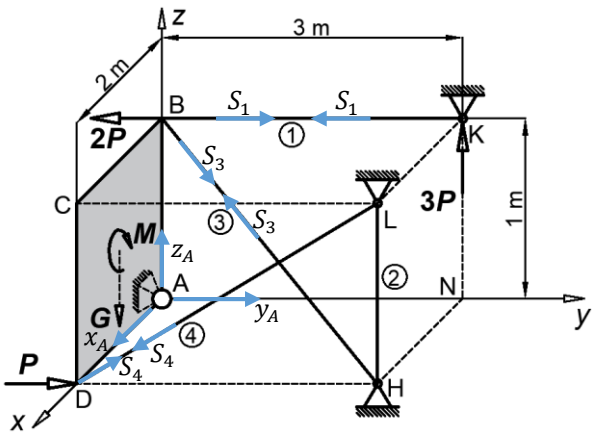
$$\vec{S}_3 = S_3\frac{2}{\sqrt{14}}\vec{\imath} + S_3\frac{3}{\sqrt{14}}\vec{j} + S_3\frac{-1}{\sqrt{14}}\vec{k}$$

$$\vec{S}_4 = S_4\frac{3}{\sqrt{10}}\vec{j} + S_4\frac{1}{\sqrt{10}}\vec{k}$$

(1) $\ldots F_{R_x} = 0 \Rightarrow x_A + S_3\frac{2}{\sqrt{14}} = 0$

(2) $\ldots F_{R_y} = 0 \Rightarrow 3\sqrt{10} - 6\sqrt{10} + y_A + S_1 + S_3\frac{3}{\sqrt{14}} + S_4\frac{3}{\sqrt{10}} = 0$

(3) $\ldots F_{R_z} = 0 \Rightarrow -4 + z_A + S_3\frac{-1}{\sqrt{14}} + S_4\frac{1}{\sqrt{10}} = 0$



$$\vec{M} = -M\vec{j} = -\vec{j}$$

$$\vec{M}_A{}^{\vec{P}} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 0 \\ 0 & 3\sqrt{10} & 0 \end{bmatrix} = 6\sqrt{10}\vec{k}$$

$$\vec{M}_A{}^{2\vec{P}} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 0 & -6\sqrt{10} & 0 \end{bmatrix} = 6\sqrt{10}\vec{i}$$

$$\vec{M}_A{}^{\vec{G}} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0{,}5 \\ 0 & 0 & -4 \end{bmatrix} = 4\vec{j}$$

$$\vec{M}_A{}^{\vec{R}_A} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0 \\ x_A & y_A & z_A \end{bmatrix} = \vec{0}$$

$$\vec{M}_A{}^{\vec{S}_1} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 0 & S_1 & 0 \end{bmatrix} = -S_1\vec{i}$$

$$\vec{M}_A{}^{\vec{S}_3} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ S_3\dfrac{2}{\sqrt{14}} & S_3\dfrac{3}{\sqrt{14}} & S_3\dfrac{-1}{\sqrt{14}} \end{bmatrix} = -S_3\frac{3}{\sqrt{14}}\vec{i} + S_3\frac{2}{\sqrt{14}}\vec{j}$$

$$\vec{M}_A{}^{\vec{S}_4} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 0 \\ 0 & S_4\dfrac{3}{\sqrt{10}} & S_4\dfrac{1}{\sqrt{10}} \end{bmatrix} = -S_4\frac{2}{\sqrt{10}}\vec{j} + S_4\frac{6}{\sqrt{10}}\vec{k}$$

(4) $\dots M_{R_x} = 0 \Rightarrow 6\sqrt{10} - S_1 - S_3\dfrac{3}{\sqrt{14}} = 0$

(5) $\dots M_{R_y} = 0 \Rightarrow -1 + 4 + S_3\dfrac{2}{\sqrt{14}} - S_4\dfrac{2}{\sqrt{10}} = 0$

(6) $\dots M_{R_z} = 0 \Rightarrow 6\sqrt{10} + S_4\dfrac{6}{\sqrt{10}} = 0$

$$(\mathbf{6}) \Rightarrow S_4 \frac{6}{\sqrt{10}} = -6\sqrt{10} \Rightarrow S_4 = -6\sqrt{10}\frac{\sqrt{10}}{6} \Rightarrow \boxed{\boldsymbol{S_4 = -10\ \mathbf{kN}}}$$

$$(\mathbf{5}) \Rightarrow -1 + 4 + S_3 \frac{2}{\sqrt{14}} + 10\frac{2}{\sqrt{10}} = 0 \Rightarrow S_3 \frac{2}{\sqrt{14}} = -3 - 2\sqrt{10}$$

$$\boxed{\boldsymbol{S_3 = -\frac{\sqrt{14}}{2}\left(3 + 2\sqrt{10}\right) = -17,44\ \mathbf{kN}}}$$

$$(\mathbf{4}) \Rightarrow 6\sqrt{10} - S_1 + \frac{\sqrt{14}}{2}\left(3 + 2\sqrt{10}\right)\frac{3}{\sqrt{14}} = 0 \Rightarrow 6\sqrt{10} - S_1 + \frac{9}{2} + 3\sqrt{10} = 0$$

$$\boxed{\boldsymbol{S_1 = \frac{9}{2} + 9\sqrt{10} = 32,96\ \mathbf{kN}}}$$

$$(\mathbf{1}) \Rightarrow x_A - \frac{\sqrt{14}}{2}\left(3 + 2\sqrt{10}\right)\frac{2}{\sqrt{14}} = 0 \Rightarrow \boxed{\boldsymbol{x_A = 3 + 2\sqrt{10} = 9,32\ \mathbf{kN}}}$$

$$(\mathbf{2}) \Rightarrow 3\sqrt{10} - 6\sqrt{10} + y_A + \frac{9}{2} + 9\sqrt{10} - \frac{\sqrt{14}}{2}\left(3 + 2\sqrt{10}\right)\frac{3}{\sqrt{14}} - 10\frac{3}{\sqrt{10}} = 0$$

$$-3\sqrt{10} + y_A + \frac{9}{2} + 9\sqrt{10} - \frac{3}{2}\left(3 + 2\sqrt{10}\right) - 3\sqrt{10} = 0$$

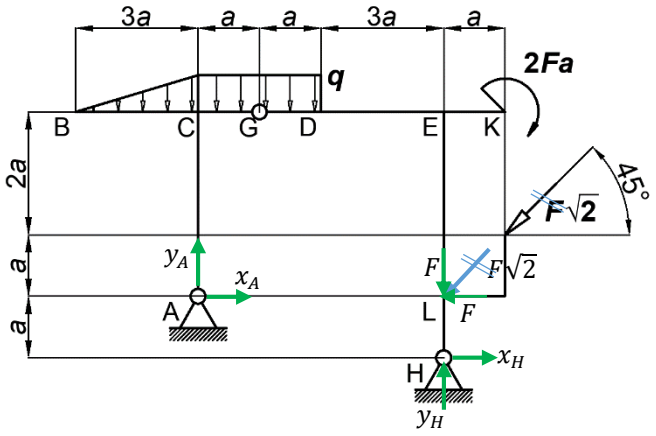
$$-3\sqrt{10} + y_A + \frac{9}{2} + 9\sqrt{10} - \frac{9}{2} - 3\sqrt{10} - 3\sqrt{10} = 0$$

$$\boxed{\boldsymbol{y_A = 0}}$$

$$(\mathbf{3}) \Rightarrow -4 + z_A + \frac{\sqrt{14}}{2}\left(3 + 2\sqrt{10}\right)\frac{1}{\sqrt{14}} - 10\frac{1}{\sqrt{10}} = 0$$

$$-4 + z_A + \frac{3}{2} + \sqrt{10} - \sqrt{10} = 0 \Rightarrow z_A = 4 - \frac{3}{2} \Rightarrow \boxed{\boldsymbol{z_A = \frac{5}{2} = 2,5\ \mathbf{kN}}}$$

ДРУГИ ЗАДАТАК



$$M_G^l = 0 \Rightarrow y_A \cdot a - x_A \cdot 3a - q \cdot a \cdot \frac{a}{2} - \frac{1}{2} q \cdot 3a \cdot (a + a) = 0 \Rightarrow y_A - 3x_A - 3{,}5qa = 0 \ldots (1)$$

$$\sum M_H = 0 \Rightarrow y_A \cdot 5a + x_A \cdot a - \frac{1}{2} q \cdot 3a \cdot (a + 5a) - q \cdot 2a \cdot 4a + 2Fa - F \cdot a = 0$$

$$5y_A + x_A - 9qa - 8qa + Fa = 0 \ldots (2)$$

$$(2) \cdot 3 + (1) \Rightarrow y_A - 3x_A - 3{,}5qa + 15y_A + 3x_A - 27qa - 24qa + 3Fa = 0$$

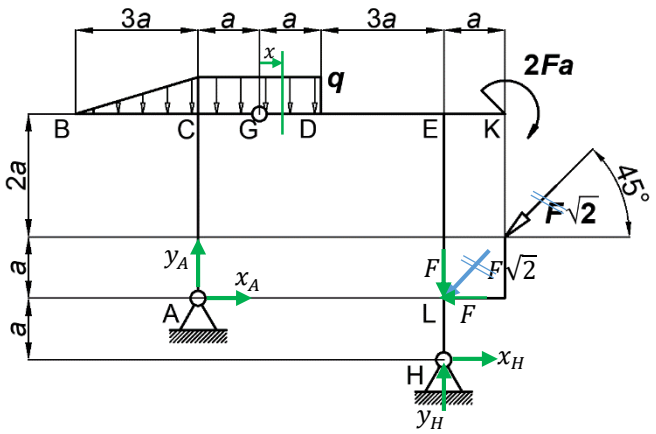
$$16y_A - 54{,}5qa + 3Fa = 0 \Rightarrow \boldsymbol{y_A} = \frac{54{,}5qa - 3Fa}{16} = \frac{109 - 12}{16} = \mathbf{6{,}0625\ kN}$$

$$(2) \Rightarrow 5 \cdot 6{,}0625 + x_A - 18 - 16 + 4 = 0 \Rightarrow \boldsymbol{x_A} = \mathbf{-0{,}3125\ kN}$$

$$\sum x_i = 0 \Rightarrow x_A - F + x_H = 0 \Rightarrow \boldsymbol{x_H} = F - x_A = \mathbf{4{,}3125\ kN}$$

$$\sum y_i = 0 \Rightarrow y_A - \frac{1}{2} q \cdot 3a - q \cdot 2a - F + y_H = 0$$

$$6{,}0625 - 3 - 4 - 4 + y_H = 0 \Rightarrow \boldsymbol{y_H} = \mathbf{4{,}9375\ kN}$$



$M_A^l = 0$

$M_{C\boxed{I}}^l = -x_A \cdot 3a = 0{,}9375 \text{ kNm}$

$M_B^l = 0$

$M_{C\boxed{L}}^l = -\frac{1}{2} q \cdot 3a \cdot a = -3 \text{ kNm}$

$M_{C\boxed{D}}^l = -\frac{1}{2} q \cdot 3a \cdot a - x_A \cdot 3a = -3 + 0{,}9375 = -2{,}0625 \text{ kNm}$

$M_{C\boxed{D}}^d = -q \cdot 2a \cdot a - 2Fa - F \cdot 5a - F \cdot 3a + y_H \cdot 5a + x_H \cdot 4a$

$= -4 - 8 - 20 - 12 + 24{,}6875 + 17{,}25 = -2{,}0625 \text{ kNm}$

$M_G = 0$

$M_D^d = -2Fa - F \cdot 3a - F \cdot 3a + y_H \cdot 3a + x_H \cdot 4a = -32 + 14{,}8125 + 17{,}25 = 0{,}0625 \text{ kNm}$

$M_{E\boxed{L}}^d = -2Fa - F \cdot 3a + x_H \cdot 4a = -20 + 17{,}25 = -2{,}75 \text{ kNm}$

$M_{E\boxed{D}}^d = -2Fa = -8 \text{ kNm}$

$M_{K\boxed{L}}^d = -2Fa = -8 \text{ kNm}$

$M_{K\boxed{D}}^d = 0$

$M_{E\boxed{I}}^d = -F \cdot 3a + x_H \cdot 4a = -12 + 17{,}25 = 5{,}25 \text{ kNm}$

$M_L^d = x_H \cdot a = 4{,}1325 \text{ kNm}$

$M_H^d = 0$

G – D

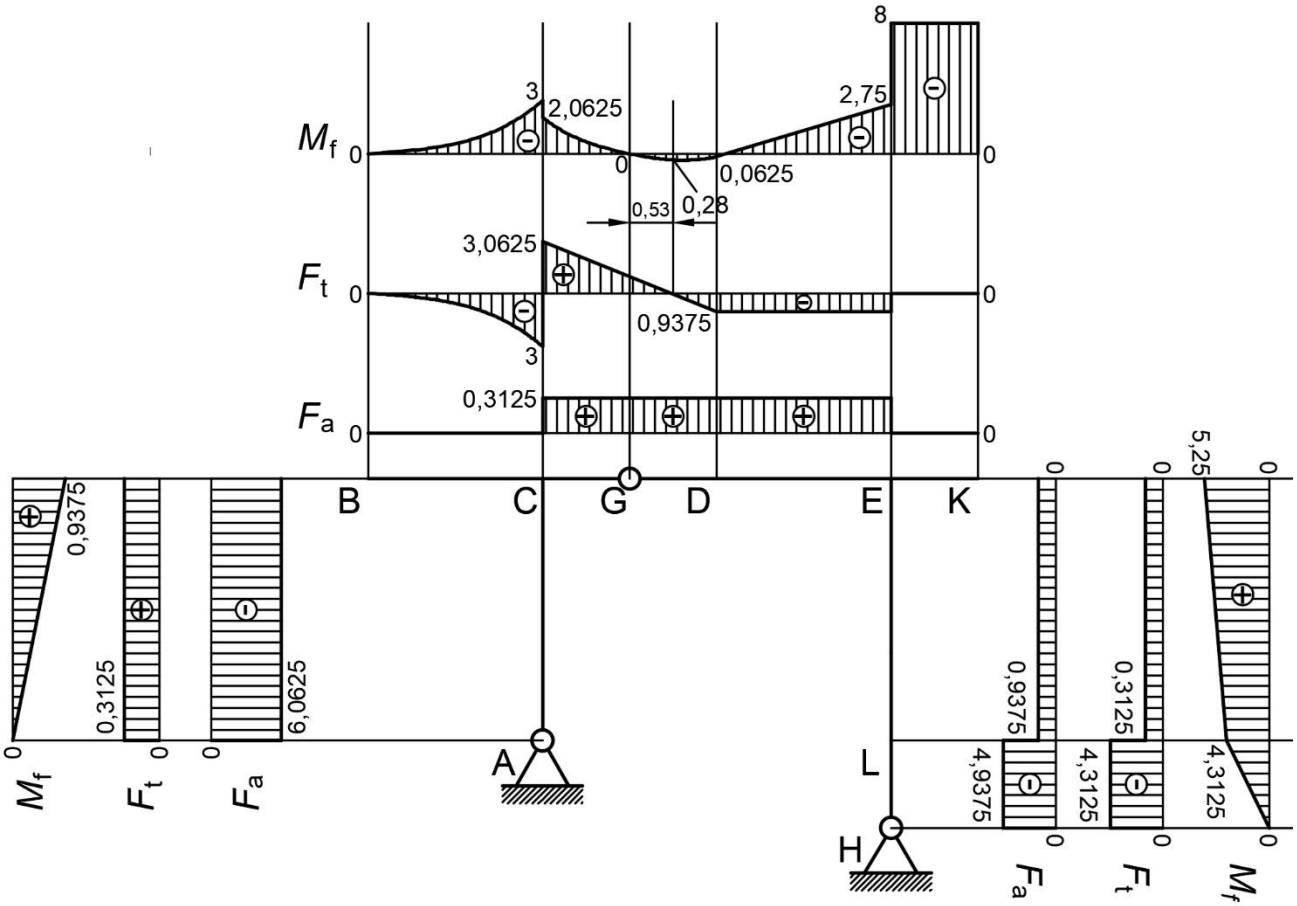
$$\boldsymbol{F_a} = -x_A = \mathbf{0{,}3125}$$

$$\boldsymbol{F_t} = y_A - \frac{1}{2} q \cdot 3a - q \cdot (a + x) = 6{,}0625 - 3 - 2 - 2x = \mathbf{1{,}0625 - 2x}$$

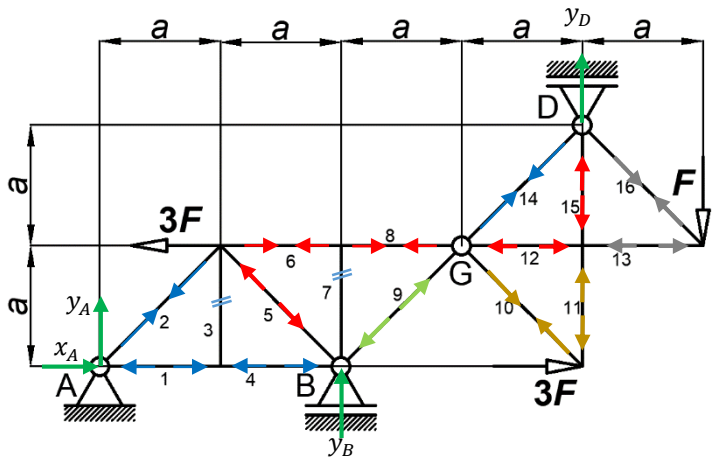
$$M_f = y_A \cdot (a + x) - x_A \cdot 3a - \frac{1}{2} q \cdot 3a \cdot (a + a + x) - q \cdot (a + x) \cdot \frac{(a + x)}{2}$$

$$\boldsymbol{M_f} = 6{,}0625 + 6{,}0625x + 0{,}9375 - 6 - 3x - a^2 - 2ax - x^2 = \mathbf{-x^2 + 1{,}0625x}$$

$$F_t^* = 0 \Rightarrow 1{,}0625 - 2x^* = 0 \Rightarrow \boldsymbol{x^*} = \mathbf{0{,}53\ m}, \qquad \boldsymbol{M_f^*} = -x^{*2} + 1{,}0625x^* = \mathbf{0{,}28\ kNm}$$


8
3
2,0625
2,75
M_f
0
0
0,0625
0,53
0,28
3,0625
F_t
0
0
0,9375
3
0,3125
F_a
0
0
B
C
G
D
E
K
5,25
0
0
0,9375
0,3125
0,9375
6,0625
0,3125
A
L
4,9375
4,3125
4,3125
H
0
0
0
0
0
0
M_f
F_t
F_a
F_a
F_t
M_f

ТРЕЋИ ЗАДАТАК

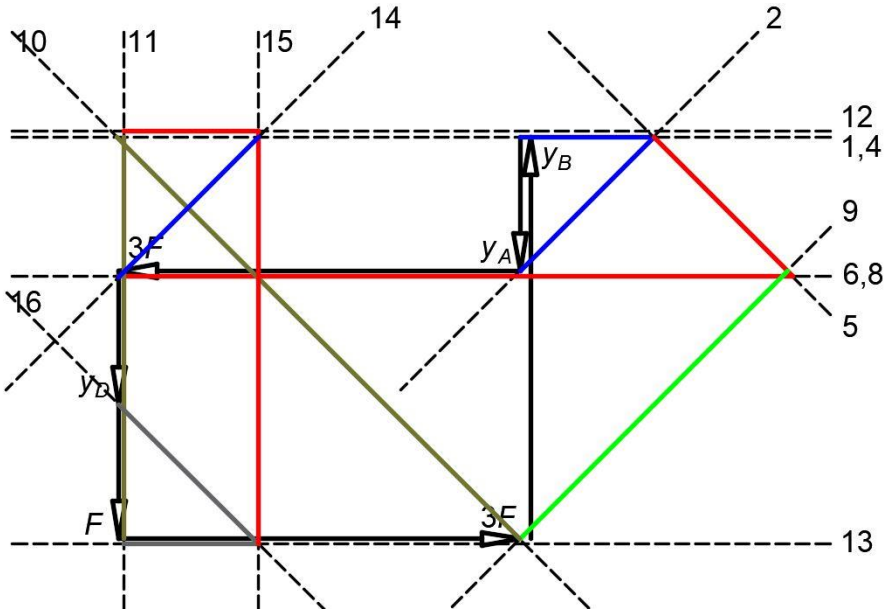


$$M_G^d = 0 \Rightarrow 3F \cdot a - F \cdot 2a + y_D \cdot a = 0 \Rightarrow \boldsymbol{y_D = -F}$$

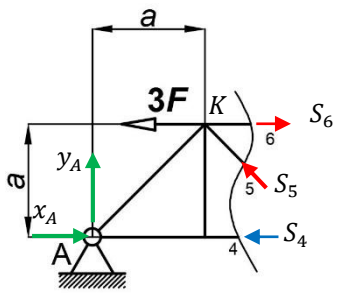
$$\sum M_A = 0 \Rightarrow 3F \cdot a + y_B \cdot 2a + y_D \cdot 4a - F \cdot 5a = 0 \Rightarrow \boldsymbol{y_B = 3F}$$

$$\sum x_i = 0 \Rightarrow x_A - 3F + 3F = 0 \Rightarrow \boldsymbol{x_A = 0}$$

$$\sum y_i = 0 \Rightarrow y_A + y_B + y_D - F = 0 \Rightarrow \boldsymbol{y_A = -F}$$



	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
И		$F\sqrt{2}$	0			$5F$	0	$5F$		$3F\sqrt{2}$				$F\sqrt{2}$		$F\sqrt{2}$
П	F			F	$F\sqrt{2}$				$2F\sqrt{2}$		$3F$	F	F		$3F$	

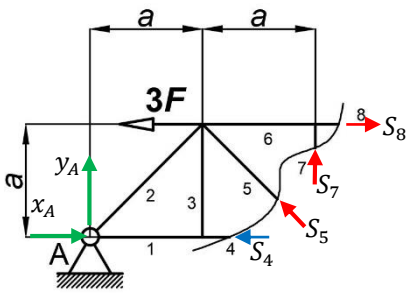


$$\sum M_K = 0 \Rightarrow y_A \cdot a - x_A \cdot a + S_4 \cdot a = 0$$

$$\boldsymbol{S_4 = -y_A = F}$$

$$\sum y_i = 0 \Rightarrow y_A + S_5 \frac{\sqrt{2}}{2} = 0$$

$$\boldsymbol{S_5 = -y_A\sqrt{2} = F\sqrt{2}}$$



$$\sum x_i = 0 \Rightarrow x_A - 3F - S_4 - S_5 \frac{\sqrt{2}}{2} + S_8 = 0$$

$$\boldsymbol{S_8 = 3F + F + F\sqrt{2}\frac{\sqrt{2}}{2} = 5F}$$

$$\sum y_i = 0 \Rightarrow y_A + S_5 \frac{\sqrt{2}}{2} + S_7 = 0$$

$$\boldsymbol{S_7 = F - F\sqrt{2}\frac{\sqrt{2}}{2} = 0}$$