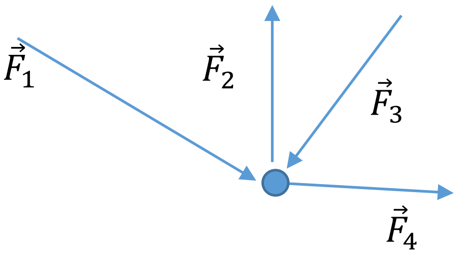
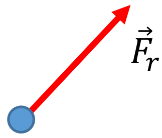
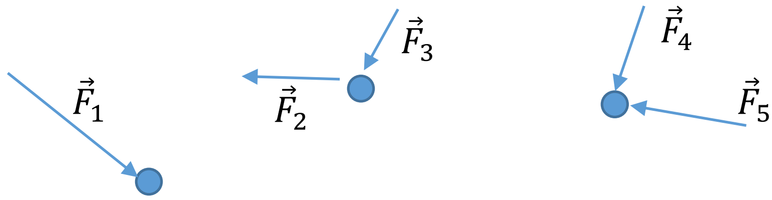
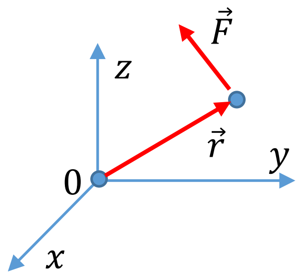




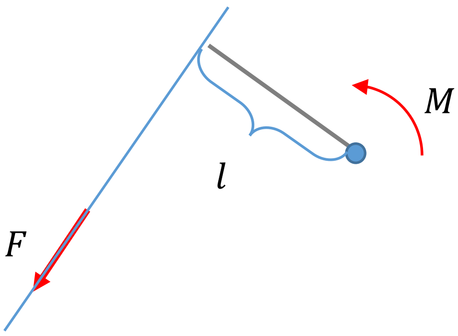
$$\vec{F}_r = F_{r_x}\vec{\imath} + F_{r_y}\vec{\jmath} + F_{r_z}\vec{k}$$



Момент (~~моменАт~~) силе за тачку

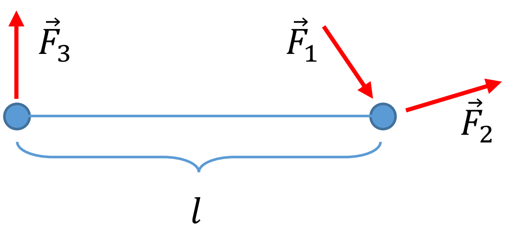


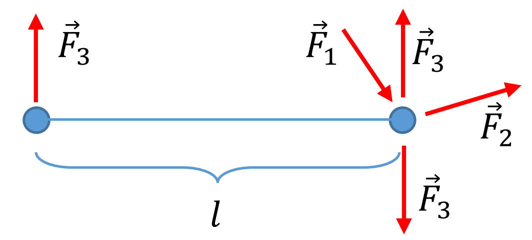
$$\vec{M}_0 = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

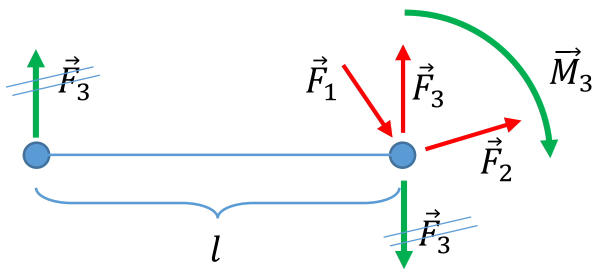


$$M = Fl$$

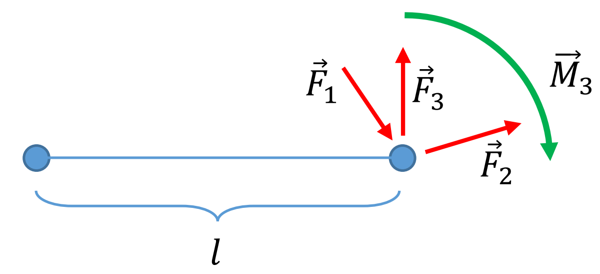
l је најкраће растојање од правца силе F до тачке за коју рачунамо момент.

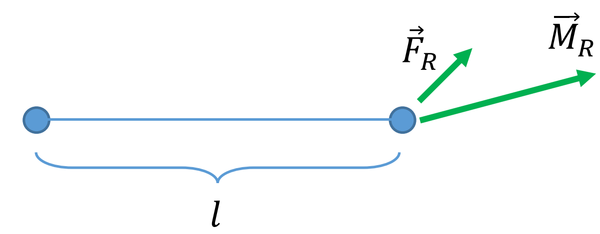






$$M_3 = F_3 l$$





$\vec{F}_r$ – резултанта

$\vec{F}_R$ – главни вектор

$\vec{M}_R$ – главни момент

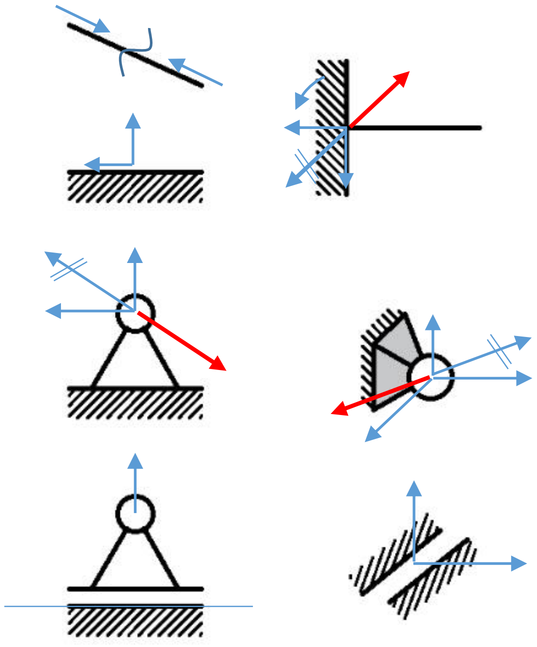
Услов равнотеже система сучељних сила:

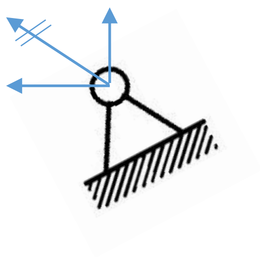
$$\vec{F}_r = \vec{0} \Rightarrow \begin{cases} F_{r_x} = 0 \\ F_{r_y} = 0 \\ F_{r_z} = 0 \end{cases}$$

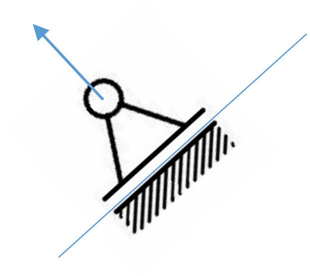
Услови равнотеже система сила који НИЈЕ сучељни:

$$\vec{F}_R = \vec{0} \Rightarrow \begin{cases} F_{R_x} = 0 \\ F_{R_y} = 0 \\ F_{R_z} = 0 \end{cases}, \qquad \vec{M}_R = \vec{0} \Rightarrow \begin{cases} M_{R_x} = 0 \\ M_{R_y} = 0 \\ M_{R_z} = 0 \end{cases}$$

Принцип ослобађања од веза

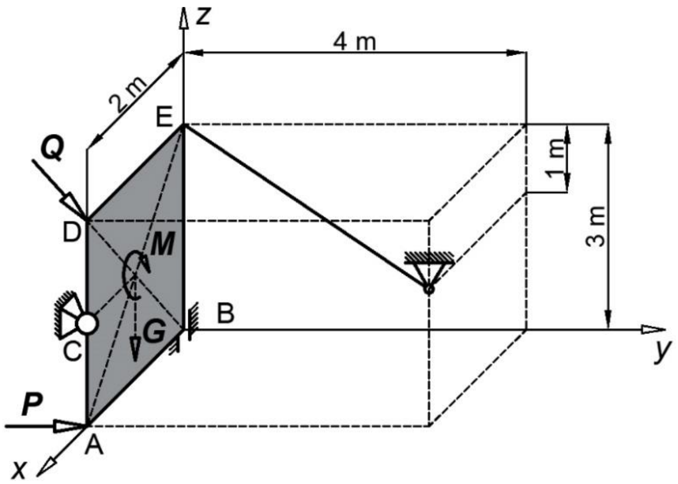


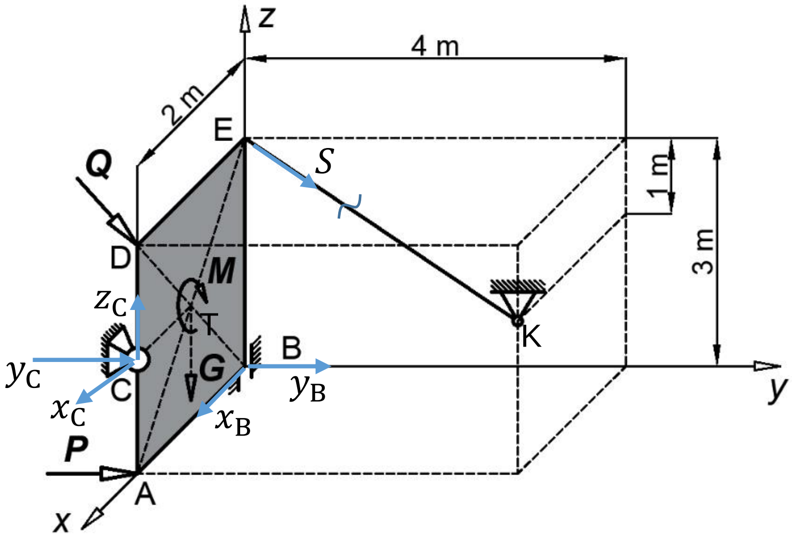




ЗАДАТАК БРОЈ 1

Одредити реакције веза хомогене плоче тежине $G = 2\,\mathrm{kN}$ приказане на слици. На плочу дјелују сила P интензитета $4\,\mathrm{kN}$ и сила Q интензитета $3\,\mathrm{kN}$. У равни плоче дјелује момент M интензитета $6\,\mathrm{kNm}$, чији је смјер дејства приказан на слици. Плоча је у тачки C везана за сферни зглоб, у тачки B за цилиндрично лежиште, а у тачки E за лаки крути штап.





Силе

$$\vec{R}_\mathrm{C} = x_\mathrm{C}\vec{\imath} + y_\mathrm{C}\vec{\jmath} + z_\mathrm{C}\vec{k}$$

$$\vec{R}_\mathrm{B} = x_\mathrm{B}\vec{\imath} + y_\mathrm{B}\vec{\jmath}$$

$$\vec{S} = S\vec{e}_\mathrm{EK} = S\frac{\overrightarrow{\mathrm{EK}}}{\mathrm{EK}} = S\frac{2\vec{\imath} + 4\vec{\jmath} - \vec{k}}{\sqrt{4+16+1}} = \frac{2S}{\sqrt{21}}\vec{\imath} + \frac{4S}{\sqrt{21}}\vec{\jmath} - \frac{S}{\sqrt{21}}\vec{k}$$

$$\vec{P} = P\vec{\jmath} = 4\vec{\jmath}$$

$$\vec{G} = -G\vec{k} = -2\vec{k}$$

$$\vec{Q} = Q\vec{e}_Q = 3\frac{\overrightarrow{\mathrm{DB}}}{\mathrm{DB}} = 3\frac{-2\vec{\imath} - 3\vec{k}}{\sqrt{4+9}} = -\frac{6}{\sqrt{13}}\vec{\imath} - \frac{9}{\sqrt{13}}\vec{k}$$

Моменти

$$\vec{M} = -M\vec{j} = -6\vec{j}$$

$$\vec{M}_{\mathrm{B}}^{\vec{R}_{\mathrm{C}}} = \vec{r}_{\mathrm{C}} \times \vec{R}_{\mathrm{C}} = \overrightarrow{\mathrm{BC}} \times \vec{R}_{\mathrm{C}} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 0 & 1{,}5 \\ x_{\mathrm{C}} & y_{\mathrm{C}} & z_{\mathrm{C}} \end{vmatrix}$$

$$= \vec{\imath}\begin{vmatrix} 0 & 1{,}5 \\ y_{\mathrm{C}} & z_{\mathrm{C}} \end{vmatrix} - \vec{\jmath}\begin{vmatrix} 2 & 1{,}5 \\ x_{\mathrm{C}} & z_{\mathrm{C}} \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 0 \\ x_{\mathrm{C}} & y_{\mathrm{C}} \end{vmatrix}$$

$$= \vec{\imath}(0 \cdot z_{\mathrm{C}} - 1{,}5 \cdot y_{\mathrm{C}}) - \vec{\jmath}(2 \cdot z_{\mathrm{C}} - 1{,}5 \cdot x_{\mathrm{C}}) + \vec{k}(2 \cdot y_{\mathrm{C}} - 0 \cdot x_{\mathrm{C}})$$

$$= -1{,}5y_{\mathrm{C}}\vec{\imath} + (1{,}5x_{\mathrm{C}} - 2z_{\mathrm{C}})\vec{\jmath} + 2y_{\mathrm{C}}\vec{k}$$

$$\vec{M}_{\mathrm{B}}^{\vec{R}_{\mathrm{B}}} = \vec{r}_{\mathrm{B}} \times \vec{R}_{\mathrm{B}} = \overrightarrow{\mathrm{BB}} \times \vec{R}_{\mathrm{B}} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & 0 & 0 \\ x_{\mathrm{B}} & y_{\mathrm{B}} & 0 \end{vmatrix} = 0\vec{\imath} + 0\vec{\jmath} + 0\vec{k} = \vec{0}$$

$$\vec{M}_{\mathrm{B}}^{\vec{S}} = \vec{r}_{\mathrm{E}} \times \vec{S} = \overrightarrow{\mathrm{BE}} \times \vec{S} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & 0 & 3 \\ \frac{2S}{\sqrt{21}} & \frac{4S}{\sqrt{21}} & -\frac{S}{\sqrt{21}} \end{vmatrix} = -\frac{12S}{\sqrt{21}}\vec{\imath} - \vec{\jmath}\left(-\frac{6S}{\sqrt{21}}\right)$$

$$= -\frac{12S}{\sqrt{21}}\vec{\imath} + \frac{6S}{\sqrt{21}}\vec{\jmath}$$

$$\vec{M}_{\mathrm{B}}^{\vec{P}} = \vec{r}_{\mathrm{A}} \times \vec{P} = \overrightarrow{\mathrm{BA}} \times \vec{P} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix} = 8\vec{k}$$

$$\vec{M}_{\mathrm{B}}^{\vec{G}} = \vec{r}_{\mathrm{T}} \times \vec{G} = \overrightarrow{\mathrm{BT}} \times \vec{G} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 0 & 1{,}5 \\ 0 & 0 & -2 \end{vmatrix} = 2\vec{\jmath}$$

$$\vec{M}_{\mathrm{B}}^{\vec{Q}} = \vec{r}_{\mathrm{D}} \times \vec{Q} = \overrightarrow{\mathrm{BD}} \times \vec{Q} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 0 & 3 \\ -\frac{6}{\sqrt{13}} & 0 & -\frac{9}{\sqrt{13}} \end{vmatrix} = -\vec{\jmath}\left(-\frac{18}{\sqrt{13}} - \left(-\frac{18}{\sqrt{13}}\right)\right) = \vec{0}$$

<u>Услови равнотеже</u>

$$\vec{M}_R = \vec{0} \Rightarrow \begin{cases} M_{R_x} = 0 \Rightarrow -1{,}5\boxed{y_{\mathrm{C}}} - \dfrac{12\boxed{S}}{\sqrt{21}} = 0 \dots (1) \\ M_{R_y} = 0 \Rightarrow -6 + 1{,}5\boxed{x_{\mathrm{C}}} - 2\boxed{z_{\mathrm{C}}} + \dfrac{6\boxed{S}}{\sqrt{21}} + 2 = 0 \dots (2) \\ M_{R_z} = 0 \Rightarrow 2\boxed{y_{\mathrm{C}}} + 8 = 0 \dots (3) \end{cases}$$

$$\vec{F}_R = \vec{0} \Rightarrow \begin{cases} F_{R_x} = 0 \Rightarrow \boxed{x_{\mathrm{C}}} + \boxed{x_{\mathrm{B}}} + \dfrac{2\boxed{S}}{\sqrt{21}} - \dfrac{6}{\sqrt{13}} = 0 \dots (4) \\ F_{R_y} = 0 \Rightarrow \boxed{y_{\mathrm{C}}} + \boxed{y_{\mathrm{B}}} + \dfrac{4\boxed{S}}{\sqrt{21}} + 4 = 0 \dots (5) \\ F_{R_z} = 0 \Rightarrow \boxed{z_{\mathrm{C}}} - \dfrac{\boxed{S}}{\sqrt{21}} - 2 - \dfrac{9}{\sqrt{13}} = 0 \dots (6) \end{cases}$$

$$(3) \Rightarrow 2y_{\mathrm{C}} + 8 = 0 \Rightarrow \boldsymbol{y_{\mathrm{C}}} = \mathbf{-4}$$

$$(1) \Rightarrow 6 - \frac{12S}{\sqrt{21}} = 0 \Rightarrow \boldsymbol{S} = \frac{\sqrt{\mathbf{21}}}{\mathbf{2}}$$

$$(5) \Rightarrow -4 + y_{\mathrm{B}} + \frac{4}{\sqrt{21}}\frac{\sqrt{21}}{2} + 4 = 0 \Rightarrow \boldsymbol{y_{\mathrm{B}}} = \mathbf{-2}$$

$$(6) \Rightarrow z_{\mathrm{C}} - \frac{1}{\sqrt{21}}\frac{\sqrt{21}}{2} - 2 - \frac{9}{\sqrt{13}} = 0 \Rightarrow \boldsymbol{z_{\mathrm{C}}} = \frac{\mathbf{5}}{\mathbf{2}} + \frac{\mathbf{9}}{\sqrt{\mathbf{13}}}$$

$$(2) \Rightarrow -6 + 1{,}5x_{\mathrm{C}} - 2\left(\frac{5}{2} + \frac{9}{\sqrt{13}}\right) + \frac{6}{\sqrt{21}}\frac{\sqrt{21}}{2} + 2 = 0 \Rightarrow \boldsymbol{x_{\mathrm{C}}} = \mathbf{4} + \frac{\mathbf{12}}{\sqrt{\mathbf{13}}}$$

$$(4) \Rightarrow 4 + \frac{12}{\sqrt{13}} + x_{\mathrm{B}} + \frac{2}{\sqrt{21}}\frac{\sqrt{21}}{2} - \frac{6}{\sqrt{13}} = 0 \Rightarrow \boldsymbol{x_{\mathrm{B}}} = \mathbf{-5} - \frac{\mathbf{6}}{\sqrt{\mathbf{13}}}$$