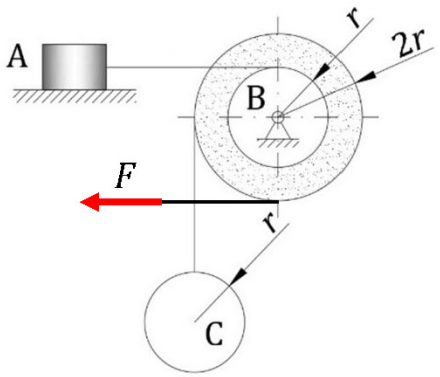
ПОПРАВНИ ДРУГОГ КОЛОКВИЈУМА ИЗ ТЕХНИЧКЕ МЕХАНИКЕ II

1. Са висине од 100 m изнад површине земље избаци се вертикално навише материјална тачка брзином $v_0 = 20$ m/s. Коликом брзином тачка пада на земљу ако је отпор ваздуха занемарљив? Колики је домет тачке ако се са исте висине избаци хоризонталном почетном брзином истог интензитета?

2. Систем приказан на слици доводи се у кретање дејством константне силе F интензитета $F = 8mg$ [N]. Коефицијент трења између тијела A и подлоге је 0,25. Одредити почетну брзину коју треба да има тијело C да би се његова брзина удвостручила након што тијело C пређе пут од 1,63 m. Дато је: $m_A = 8m, m_B = 32m, m_C = 4m, m = 500$ g и $i_B = r/2$.

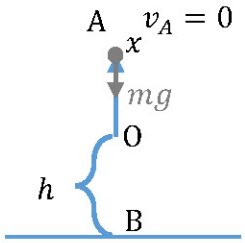


ПРВИ ЗАДАТАК

$$h = 100\ \text{m}, \qquad v_0 = 20\ \text{m/s}, \qquad v_B = ?$$

Брзина пада на земљу

I начин

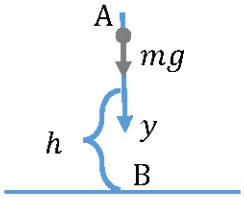


$$m\vec{a} = \vec{F} \Rightarrow ma = -mg \Rightarrow a = -g$$

$$\left.\begin{matrix} a = -g \\ a = \dfrac{dv}{dt} \end{matrix}\right\} \Rightarrow dv = -gdt \Rightarrow \int_{v_0}^{v} dv = -g\int_{0}^{t} dt \Rightarrow v = v_0 - gt$$

$$\left.\begin{matrix} v = v_0 - gt \\ v = \dfrac{dx}{dt} \end{matrix}\right\} \Rightarrow dx = (v_0 - gt)dt \Rightarrow \int_{0}^{x} dx = \int_{0}^{t} (v_0 - gt)dt \Rightarrow x = v_0 t - \frac{gt^2}{2}$$

$$\left.\begin{matrix} v_A = v_0 - gt_A \\ v_A = 0 \end{matrix}\right\} \Rightarrow t_A = \frac{v_0}{g} \Rightarrow x_A = v_0\frac{v_0}{g} - \frac{g}{2}\left(\frac{v_0}{g}\right)^2 = \frac{v_0^2}{2g}$$



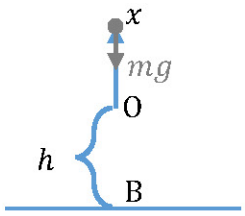
$$m\vec{a} = \vec{F} \Rightarrow ma = mg \Rightarrow a = g \Rightarrow \left.\begin{matrix} a = g \\ a = \dfrac{dv}{dt} \end{matrix}\right\} \Rightarrow dv = gdt \Rightarrow \int_{v_A}^{v} dv = g\int_{0}^{t} dt \Rightarrow v = gt$$

$$\left.\begin{matrix} v = gt \\ v = \dfrac{dy}{dt} \end{matrix}\right\} \Rightarrow dy = gtdt \Rightarrow \int_{0}^{y} dy = \int_{0}^{t} gtdt \Rightarrow y = \frac{gt^2}{2}$$

$$\left.\begin{matrix} y_B = \dfrac{gt_B^2}{2} \\ y_B = h + x_A = h + \dfrac{v_0^2}{2g} \end{matrix}\right\} \Rightarrow \frac{gt_B^2}{2} = h + \frac{v_0^2}{2g} \Rightarrow t_B = \sqrt{\frac{2h}{g} + \frac{v_0^2}{g^2}} = \sqrt{\frac{2gh + v_0^2}{g^2}} = \frac{\sqrt{v_0^2 + 2gh}}{g}$$

$$\boldsymbol{v_B} = gt_B = g\frac{\sqrt{v_0^2 + 2gh}}{g} = \sqrt{\boldsymbol{v_0^2 + 2gh}} = \sqrt{400 + 2 \cdot 9{,}81 \cdot 100} = \mathbf{48{,}6\ m/s}$$

II начин



$$m\vec{a} = \vec{F} \Rightarrow ma = -mg \Rightarrow a = -g$$

$$\left.\begin{aligned} a &= -g \\ a &= \frac{dv}{dt} \end{aligned}\right\} \Rightarrow dv = -gdt \Rightarrow \int_{v_0}^{v} dv = -g\int_{0}^{t} dt \Rightarrow v = v_0 - gt$$

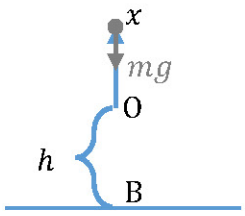
$$\left.\begin{aligned} v &= v_0 - gt \\ v &= \frac{dx}{dt} \end{aligned}\right\} \Rightarrow dx = (v_0 - gt)dt \Rightarrow \int_{0}^{x} dx = \int_{0}^{t} (v_0 - gt)dt \Rightarrow x = v_0 t - \frac{gt^2}{2}$$

$$\left.\begin{aligned} x_B &= v_0 t_B - \frac{gt_B^2}{2} \\ x_B &= -h \end{aligned}\right\} \Rightarrow v_0 t_B - \frac{gt_B^2}{2} = -h \Rightarrow \frac{gt_B^2}{2} - v_0 t_B - h = 0$$

$$t_{B\,1/2} = \frac{v_0 \pm \sqrt{v_0^2 + 2gh}}{g} \Rightarrow t_B = \frac{v_0 + \sqrt{v_0^2 + 2gh}}{g}$$

$$\boldsymbol{v_B} = v_0 - gt_B = v_0 - g\frac{v_0 + \sqrt{v_0^2 + 2gh}}{g} = \boldsymbol{\sqrt{v_0^2 + 2gh}}$$

III начин



$$m\vec{a} = \vec{F} \Rightarrow ma = -mg \Rightarrow a = -g$$

$$\left.\begin{aligned} a &= -g \\ a &= \frac{dv}{dt} = \frac{dv}{dt}\frac{dx}{dx} = \frac{vdv}{dx} \end{aligned}\right\} \Rightarrow vdv = -gdx \Rightarrow \int_{v_0}^{v_B} vdv = -g\int_{0}^{x_B=-h} dx \Rightarrow \frac{v_B^2}{2} - \frac{v_0^2}{2} = gh$$

$$\boldsymbol{v_B} = \boldsymbol{\sqrt{v_0^2 + 2gh}}$$

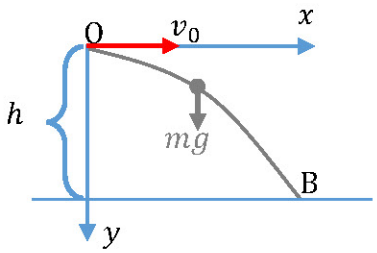
IV начин

$$E_{k_B} - E_{k_O} = A_{OB}$$

$$\frac{mv_B^2}{2} - \frac{mv_0^2}{2} = mgh$$

$$\boldsymbol{v_B} = \boldsymbol{\sqrt{v_0^2 + 2gh}}$$

Домет тачке



$$m\vec{a} = \vec{F} \Rightarrow \left.\begin{matrix} ma_x = 0 \\ ma_y = mg \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a_x = 0 \\ a_y = g \end{matrix}\right\}$$

$$\left.\begin{matrix} a_x = 0 \\ a_x = \dfrac{dv_x}{dt} \end{matrix}\right\} \Rightarrow dv_x = 0dt \Rightarrow \int_{v_0}^{v_x} dv_x = 0 \int_0^t dt \Rightarrow v_x = v_0$$

$$\left.\begin{matrix} v_x = v_0 \\ v_x = \dfrac{dx}{dt} \end{matrix}\right\} \Rightarrow dx = v_0 dt \Rightarrow \int_0^x dx = v_0 \int_0^t dt \Rightarrow x = v_0 t$$

$$\left.\begin{matrix} a_y = g \\ a_y = \dfrac{dv_y}{dt} \end{matrix}\right\} \Rightarrow dv_y = gdt \Rightarrow \int_0^{v_y} dv_y = g \int_0^t dt \Rightarrow v_y = gt$$

$$\left.\begin{matrix} v_y = gt \\ v_y = \dfrac{dy}{dt} \end{matrix}\right\} \Rightarrow dy = gtdt \Rightarrow \int_0^y dy = g \int_0^t tdt \Rightarrow y = \frac{gt^2}{2}$$

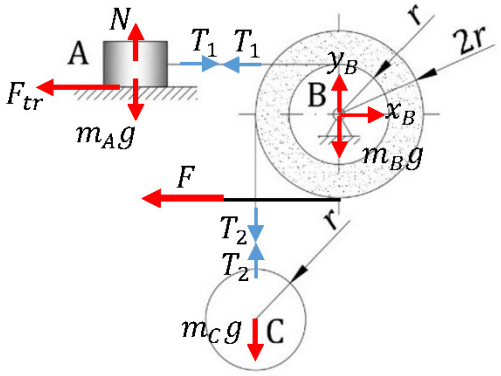
$$\left.\begin{matrix} y_B = \dfrac{gt_B^2}{2} \\ y_B = h \end{matrix}\right\} \Rightarrow \frac{gt_B^2}{2} = h \Rightarrow t_B = \sqrt{\frac{2h}{g}}$$

$$\boldsymbol{x_B = v_0 t_B = v_0 \sqrt{\frac{2h}{g}} = 20 \sqrt{\frac{200}{9{,}81}} = 90{,}30\ \mathrm{m}}$$

ДРУГИ ЗАДАТАК

$$F = 8mg, \qquad \mu = 0{,}25, \qquad v_{C\,0} = ?, \qquad v_C^* = 2v_{C\,0}, \qquad s_C^* = 1{,}63\ \text{m}$$

$$m_\text{A} = 8m, \qquad m_\text{B} = 32m, \qquad m_\text{C} = 4m, \qquad m = 500\ \text{g}, \qquad i_\text{B} = r/2$$



I начин

$$\#\ m_A\vec{a}_A = \vec{F}_A \Rightarrow \left.\begin{matrix} m_Aa_A = T_1 - F_{tr} \\ m_A \cdot 0 = N - m_Ag \end{matrix}\right\} \Rightarrow \left.\begin{matrix} m_Aa_A = T_1 - \mu m_Ag \\ N = m_Ag \end{matrix}\right\} \Rightarrow \boxed{8ma_A = T_1 - 2mg}$$

$$\#\ I_B\varepsilon_B = \sum M_B \Rightarrow m_Bi_B^2\varepsilon_B = F \cdot 2r - T_1 \cdot r - T_2 \cdot 2r \Rightarrow 32m \cdot 0{,}25r^2 \cdot \varepsilon_B = F \cdot 2r - T_1 \cdot r - T_2 \cdot 2r$$

$$\boxed{8mr\varepsilon_B = 2F - T_1 - 2T_2}$$

$$\#\ m_C\vec{a}_C = \vec{F}_C \Rightarrow m_Ca_C = T_2 - m_Cg \Rightarrow 4ma_C = T_2 - 4mg \Rightarrow \boxed{8ma_C = 2T_2 - 8mg}$$

Сабирамо три уоквирене релације:

$$8ma_A + 8mr\varepsilon_B + 8ma_C = T_1 - 2mg + 2F - T_1 - 2T_2 + 2T_2 - 8mg$$

$$8ma_A + 8mr\varepsilon_B + 8ma_C = 2F - 10mg$$

$$8ma_A + 8mr\varepsilon_B + 8ma_C = 16mg - 10mg$$

$$\boxed{8a_A + 8r\varepsilon_B + 8a_C = 6g}$$

Сва убрзања/угаона убрзања треба да се изразе само преко једног.

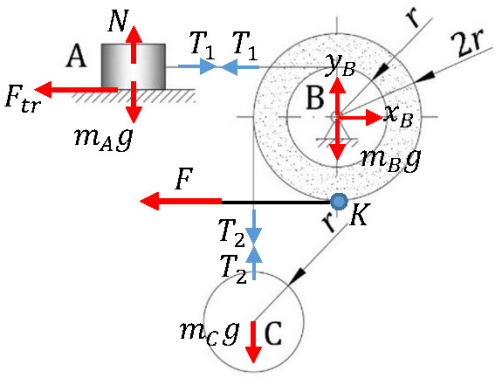
$$v_C = 2r\omega_B \Rightarrow \omega_B = \frac{v_C}{2r} \Rightarrow \varepsilon_B = \frac{a_C}{2r}$$

$$v_A = r\omega_B = r\frac{v_C}{2r} = \frac{v_C}{2} \Rightarrow a_A = \frac{a_C}{2}$$

$$4a_C + 4a_C + 8a_C = 6g \Rightarrow \boxed{a_C = \frac{3g}{8}}$$

$$\left.\begin{matrix} a_C = \dfrac{3g}{8} \\ a_C = \dfrac{dv_C}{dt} = \dfrac{dv_C}{dt}\dfrac{ds_C}{ds_C} = \dfrac{v_Cdv_C}{ds_C} \end{matrix}\right\} \Rightarrow v_Cdv_C = \frac{3g}{8}ds_C \Rightarrow \int_{v_{C\,0}}^{2v_{C\,0}} v_Cdv_C = \frac{3g}{8}\int_0^{s_C^*=1{,}63} ds_C$$

$$\frac{(2v_{C\,0})^2}{2} - \frac{v_{C\,0}^2}{2} = 1{,}63\frac{3g}{8} \Rightarrow 3v_{C\,0}^2 = 1{,}63\frac{3g}{4} \Rightarrow \boldsymbol{v_{C\,0}} = \sqrt{1{,}63\frac{g}{4}} \Rightarrow v_{C\,0} = \sqrt{1{,}63\frac{9{,}81}{4}} = \mathbf{2\ m/s}$$



II način

$$E_{k_*} - E_{k_0} = A_{0*}$$

$$E_k = \frac{m_A v_A^2}{2} + \frac{I_B \omega_B^2}{2} + \frac{m_C v_C^2}{2} = \frac{m_A \left(\frac{v_C}{2}\right)^2}{2} + \frac{m_B i_B^2 \left(\frac{v_C}{2r}\right)^2}{2} + \frac{m_C v_C^2}{2} = \frac{8m\frac{v_C^2}{4}}{2} + \frac{32m\frac{r^2}{4}\frac{v_C^2}{4r^2}}{2} + \frac{4mv_C^2}{2}$$

$$E_k = mv_C^2 + mv_C^2 + 2mv_C^2 = 4mv_C^2 \Rightarrow \left.\begin{matrix} E_{k_*} = 4mv_C^{*2} = 16m{v_C}_0^2 \\ E_{k_0} = 4m{v_C}_0^2 \end{matrix}\right\}$$

$$A_{0*}^F = \int_0^{s_C^*} \vec{F} \cdot d\vec{r}_K = \int_0^{s_C^*} F dr_K = \int_0^{s_C^*} F ds_C = \int_0^{s_C^*} 8mg ds_C = 8mgs_C^*$$

$$A_{0*}^{m_C g} = -m_C g s_C^* = -4mgs_C^*$$

$$A_{0*}^{F_{tr}} = -F_{tr} s_A^* = -F_{tr}\frac{s_C^*}{2} = -\mu m_A g \frac{s_C^*}{2} = -mgs_C^*$$

$$E_{k_*} - E_{k_0} = A_{0*}$$

$$16m{v_C}_0^2 - 4m{v_C}_0^2 = 8mgs_C^* - 4mgs_C^* - mgs_C^*$$

$$12m{v_C}_0^2 = 3mgs_C^*$$

$$\boldsymbol{v_{C_0}} = \sqrt{\frac{g}{4}s_C^*} = \sqrt{\frac{9{,}81}{4}1{,}63} = \mathbf{2\ m/s}$$